King Abdul-Aziz University

Mechanical Engineering Department

ME451
Refrigeration and Air Conditioning

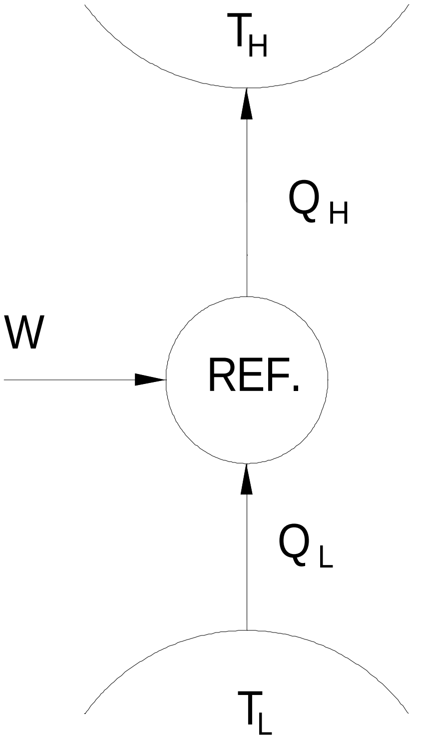
Theoretical Simple Vapor Compression Refrigeration Cycle

Contents

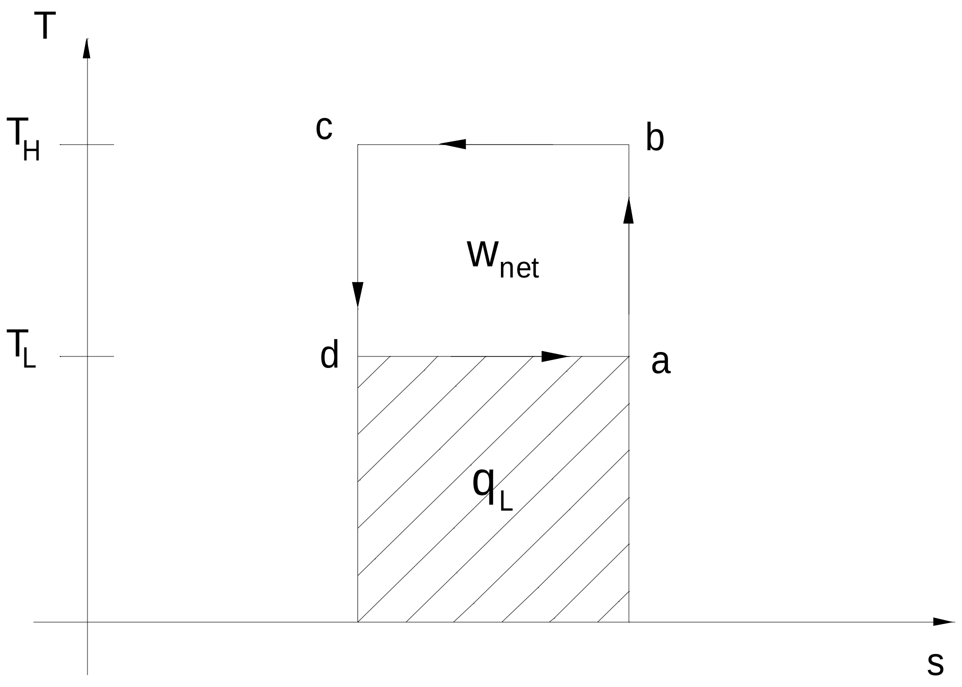
Carnot Refrigerator

$$C.O.P. = \frac{Q_L}{W} = \frac{q_L}{w}$$

$$(C.O.P)_{R,rev} = \frac{Q_L}{Q_H - Q_L} = \frac{T_L}{T_H - T_L} = \frac{1}{T_H/T_L - 1}$$

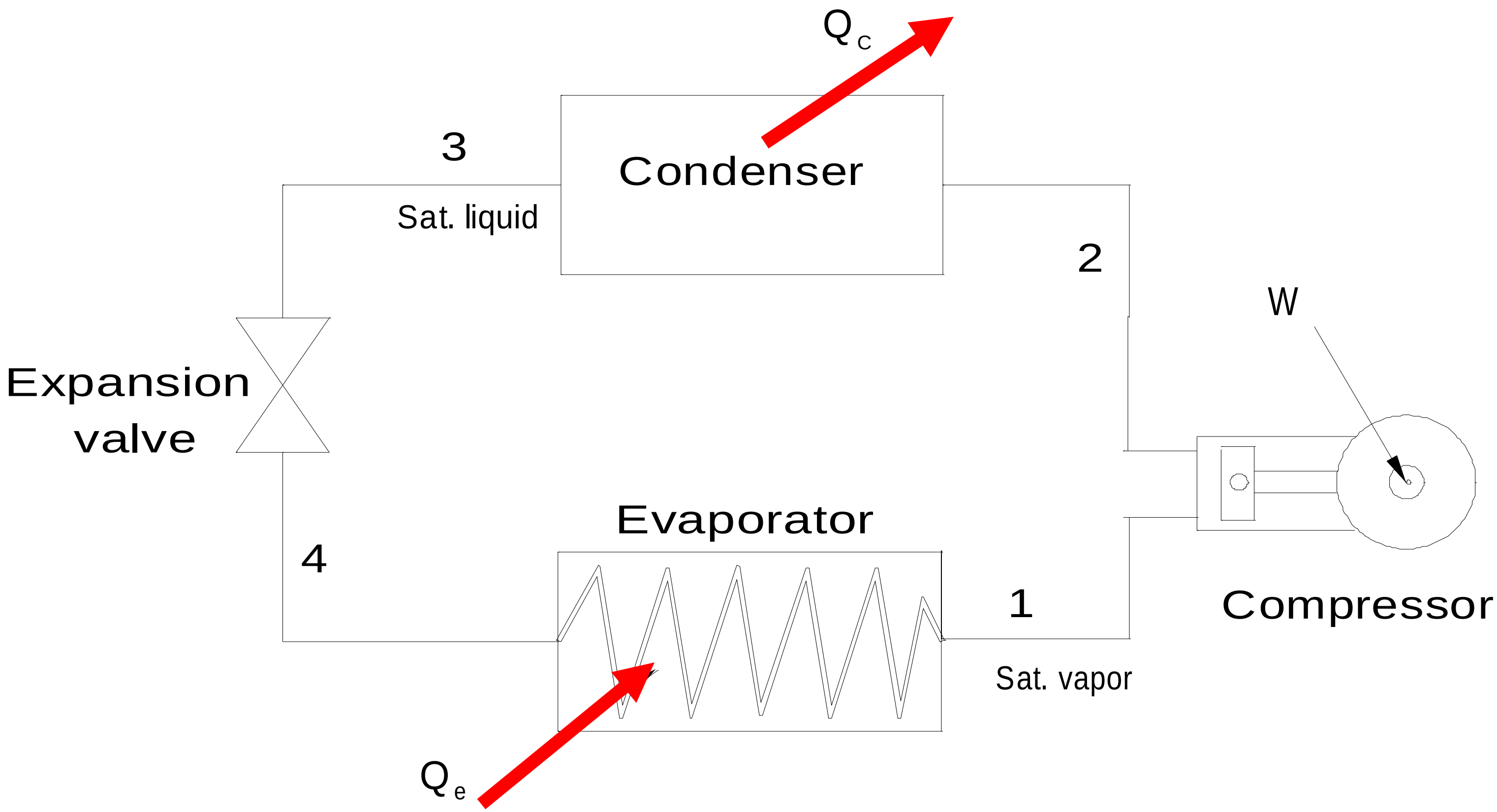


T-s diagram for the Carnot Cycle

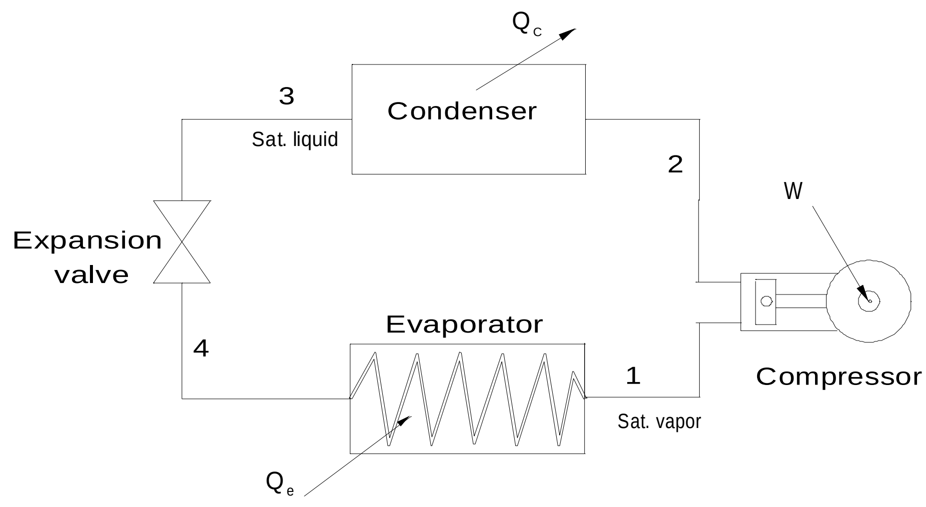


$$\left(C.O.P\right)_{Carnot} = \frac{q_L}{w_{net}} = \frac{(s_a - s_d)\,T_L}{(s_a - s_d)(T_H - T_L)} = \frac{T_L}{T_H - T_L} = \frac{1}{T_H / T_L - 1}$$

Theoretical (Simple) Vapor Compression Cycle

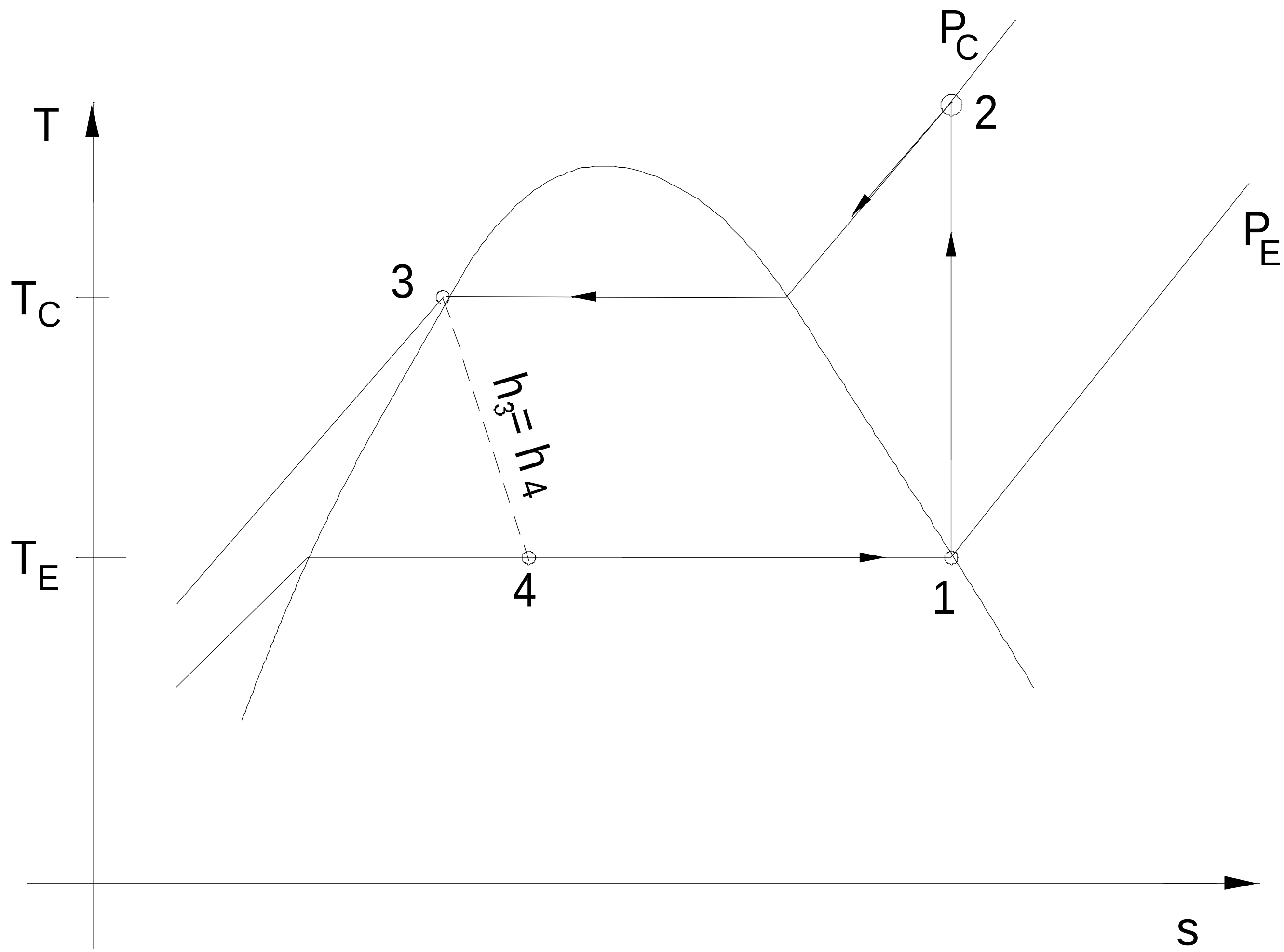


Ideal Vapor Compression Refrigeration Cycle

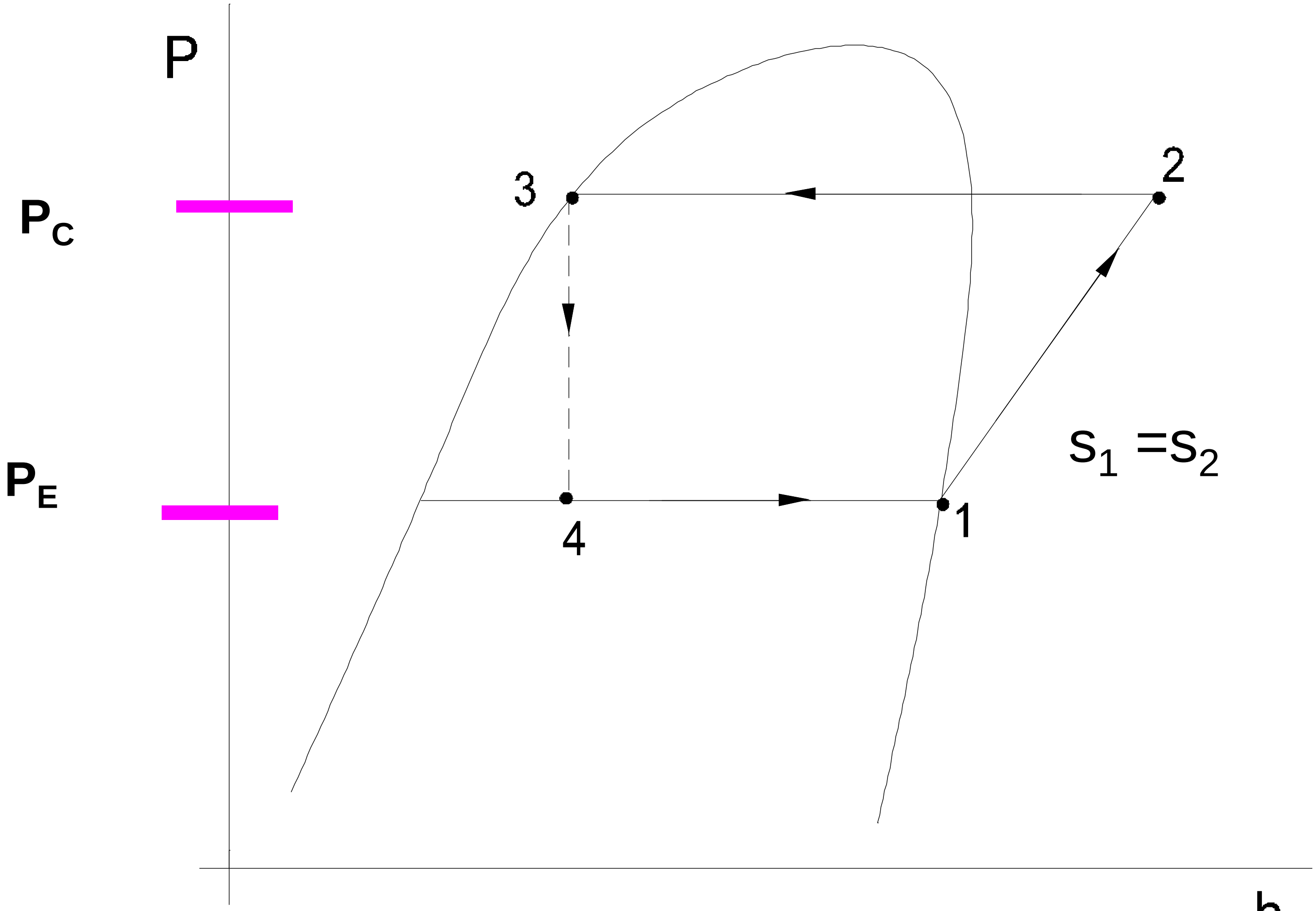


- 1 →2 Isentropic compression ($s_1=s_2$)
- 2 →3 Isobaric heat rejection ($P=P_C$)
- 3 →4 Irreversible (throttling) process ($h_3=h_4$)
- 4 →1 Isobaric heat addition (P=PE)

T-s Diagram for theoretical V.C.R. Cycle

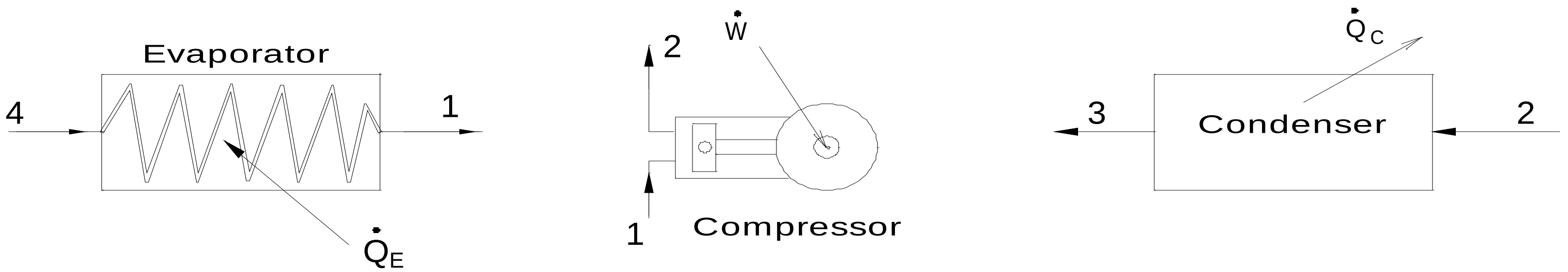


P-h diagram for Theoretical V.C.R. Cycle



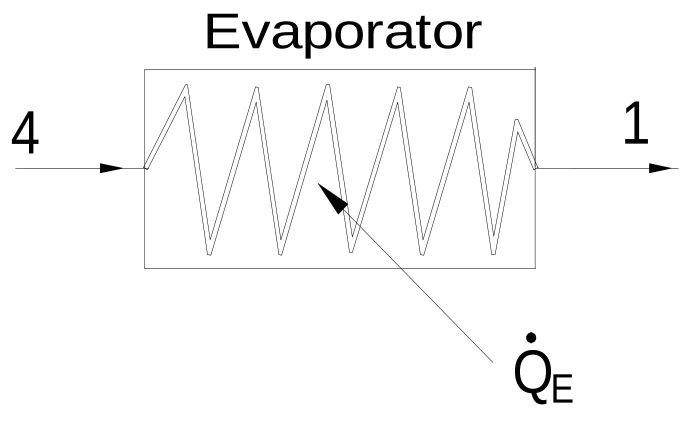
COP for the theoretical cycle

Energy Balance for each component



$$COP = \frac{\dot{Q}_e}{\dot{W}} = \frac{q_e}{w}$$

Energy balance for the evaporator



$$\dot{Q}_E = \dot{m}(h_1 - h_4)$$

or

$$q_E = (h_1 - h_4)$$

The same thing can be done for other components such as the compressor, the condenser, and the expansion valve

$$(C.O.P)_R = \frac{\dot{Q}_e}{\dot{W}} = \frac{\dot{m}\left(h_1 - h_4\right)}{\dot{m}\left(h_2 - h_1\right)} = \frac{h_1 - h_4}{h_2 - h_1}$$

Refrigeration efficiency

$$\eta_R = \frac{(C.O.P)_R}{\left(C.O.P.\right)_{R,rev}}$$

R in the subscript to indicate refrigerator not a heat pump

$$\frac{kW}{ton} = \frac{\dot{W}[kW]}{\dot{Q}_e[kW]} \cdot \frac{1}{\frac{1}{3.52}\frac{ton}{kW}} = \frac{3.52}{COP}$$

$$\frac{HP}{ton} = \frac{\dot{W}[kW]}{\dot{Q}_e[kW]} \cdot \frac{\frac{HP}{0.745kW}}{\frac{ton}{3.52kW}} = \frac{4.72}{COP}$$

EER=Energy Efficiency Ratio

$$EER = \frac{\dot{Q}_e[W]}{\dot{W}[W]} \cdot \frac{\frac{3.413 Btu/hr}{W}}{} = 3.413COP$$

EER units: Btu/h for each Watt power

Comparison between Carnot and theoretical refrigeration cycle

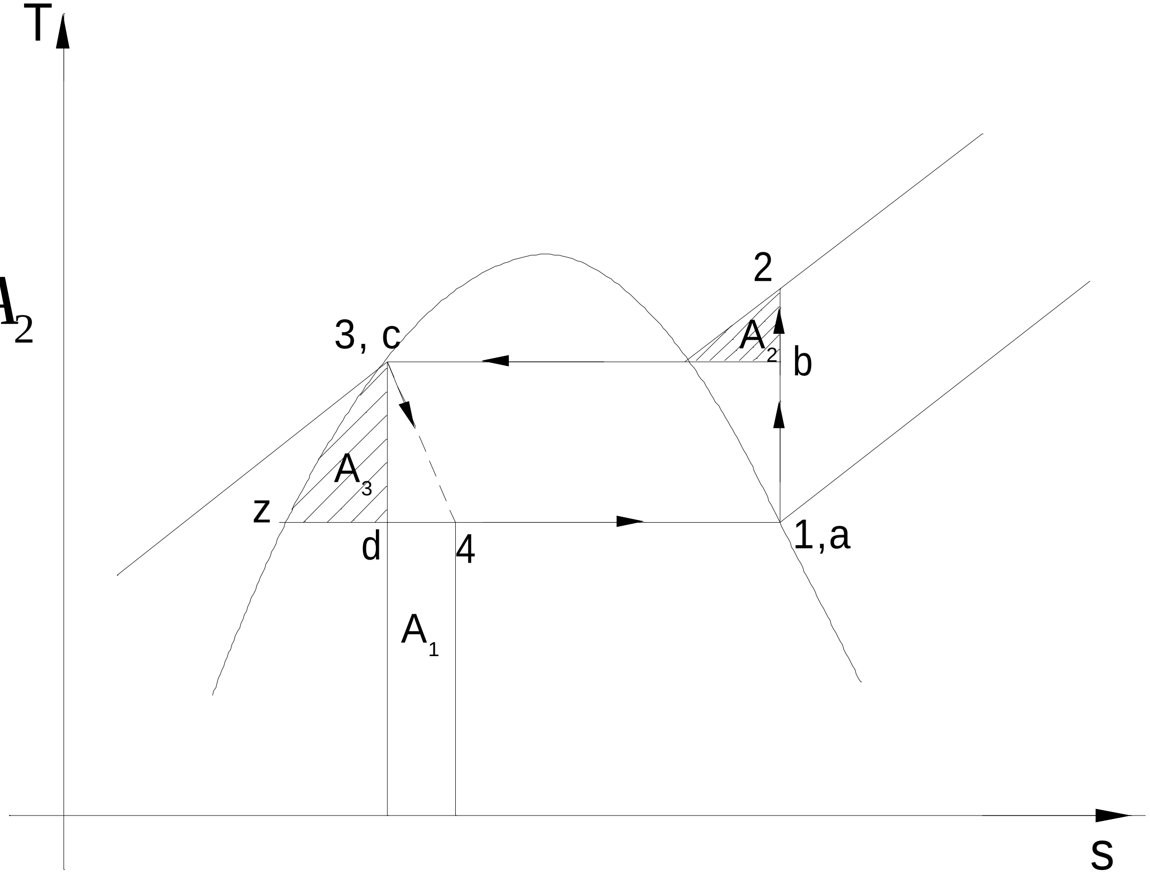
$$q_e = h_1 - h_4 = (q_e)_R - A_1$$

$$q_c = h_2 - h_3 = (q_c)_R + A_2$$

$$w = q_c - q_e$$

$$COP_R = \frac{q_e}{q_c} = \frac{(q_e)_R - A_1}{(w)_R + A_1 + A_2}$$

$$A_1 = A_3$$



Summary of properties for Ideal vapor Compression Cycle

State	T	P	h	s	v
1					Sat. V
2					
3					Sat. L
4					

Example of Ideal VCR

Given: P_E, P_C, Refrigerant type

State	T	P	h	s	v
1		P_E	h_g	$s_1=s_g$	
2		P_C		$s_2= s_1$	
3		P_C	h_f		
4		P_E	$h_4=h_3$		

Example 2.1

Consider a R-134a theoretical vapor compression machines. The condenser pressure is 1.8 MPa. Saturated refrigerant at 5 °C leaves the evaporator. Draw the cycle on P-h and T-s diagrams, and find

a-the refrigeration effect qe

b-the compressor specific work w

c-C.O.P. , $(C.O.P)_{R,rev}$ and η_R

d-Mass flow rate of refrigerant if the capacity is 10 tons

e-Repeat the calculations if Te is lowered to -25 °C.

$$(C.O.P)_{R,rev} = \frac{5+273}{62.9-5} = 4.8$$

$$\dot{m} = \frac{\dot{Q}_e}{q_e} = \frac{35.16}{108.8} = 0.323\, kg/s$$

$$COP_R = \frac{h_1 - h_4}{h_2 - h_1} = 3.21$$

$$\eta_R = \frac{COP_R}{COP_{R,rev}} = \frac{3.21}{4.8} = 0.668$$

State	T [°C]	P [kPa]	h [kJ/kg]	s [kJ/kg. K]
1	5	350.9	401.3	1.7239
2	68.3	1800	435.2	1.7239
3	62.9	1800	292.5	
4	5	350.9	292.5	

Case when T_E=-25 C

State	T [°C]	P [kPa]	h [kJ/kg]	s [kJ/kg. K]
1	-25	107.2	382.95	1.7441
2	73.64	1800	442.16	1.7441
3		1800	292.50	
4		107.2	292.50	

$$COP_R = \frac{h_1 - h_4}{h_2 - h_1} = 1.53$$

$$(C.O.P)_{R,rev} = \frac{-25 + 273}{62.9 - (-25)} = 2.82$$

$$\eta_R = \frac{(C.O.P)_R}{(C.O.P.)_{R,rev}} = 0.54$$

$$\dot{m} = \frac{\dot{Q}_e}{q_e} = \frac{35.160}{90.45} = 0.389 \text{ kg/s}$$

Effect of changing T_E and T_C on the performance of the vapor compression cycle

We know for the ideal Carnot refrigeration cycle (using either the T-s diagram or the following relation

$$(C.O.P)_{R,rev} = \frac{Q_L}{Q_H - Q_L} = \frac{T_L}{T_H - T_L} = \frac{1}{T_H / T_L - 1}$$

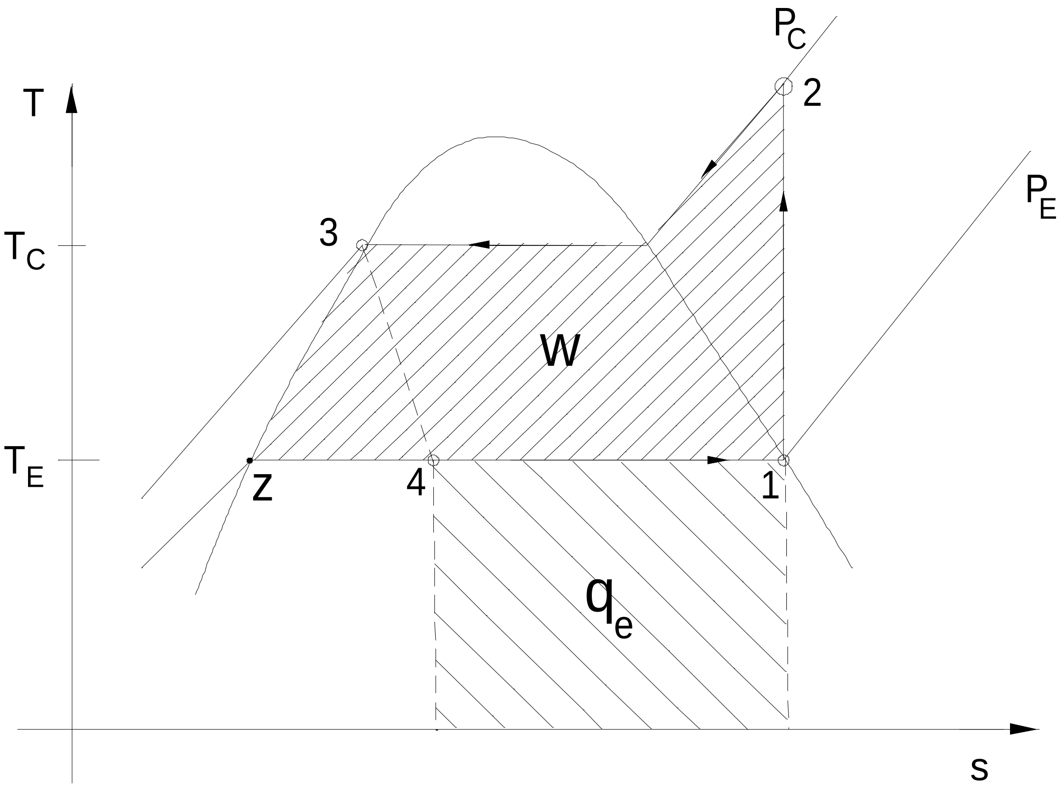
As T_E is lowered while T_C is fixed, the COP_R decreases

This is true also for the theoretical vapor compression cycle as will be shown in the next slide

q_E is given by the area as shown.

The work is

$$w = h_2 - h_1$$



$$w = h_2 - h_1 + h_z - h_z = (h_2 - h_z) - (h_1 - h_z)$$

Which is given by the indicated area

It is clear then that as T_L is lowered, q_e decreases, and w increases which means that the COP decreases

Using P-h chart

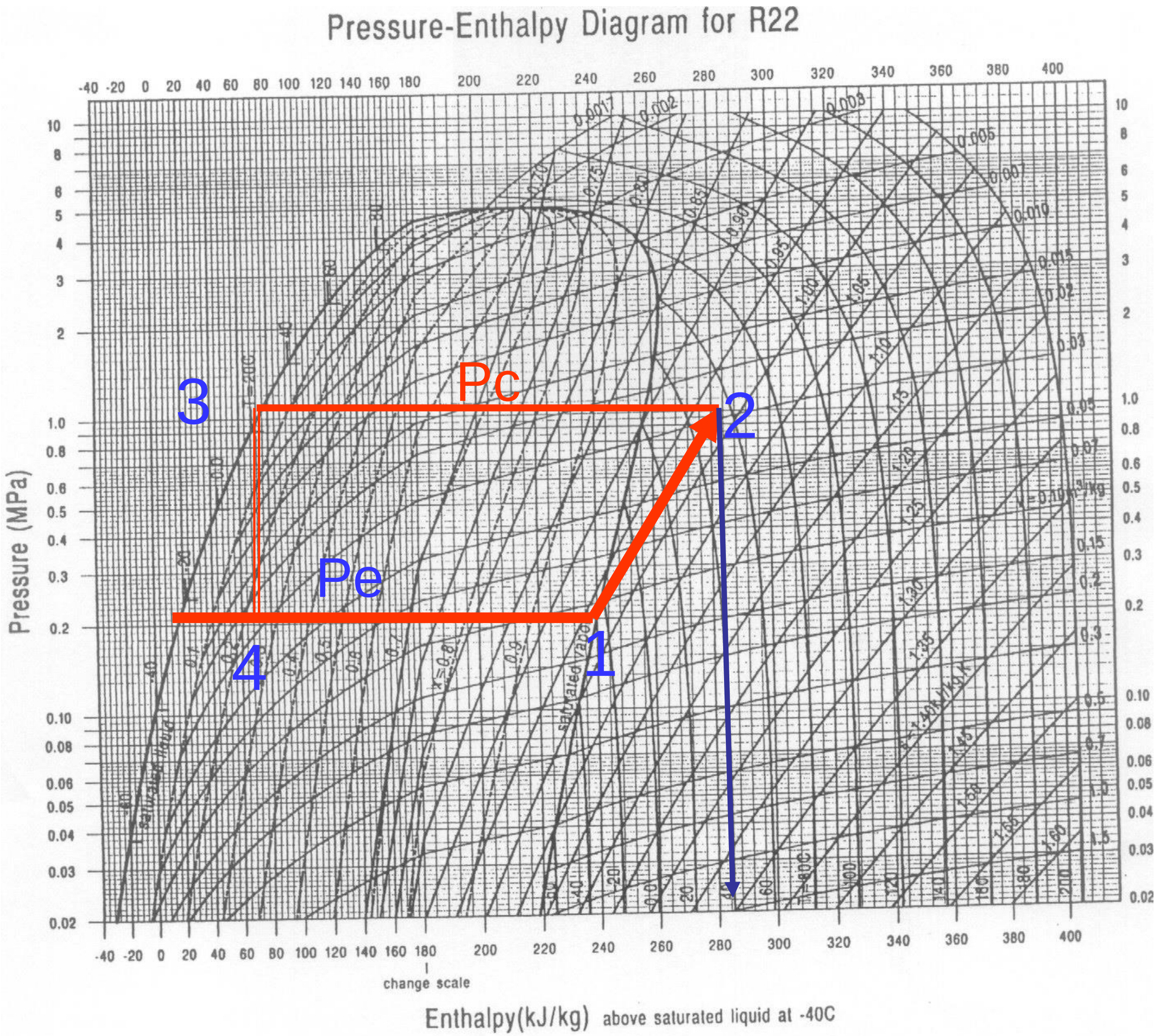

Pressure-Enthalpy Diagram for R22
Pressure (MPa)
Enthalpy(kJ/kg) above saturated liquid at -40C
change scale
Pc
Pe
1
2
3
4
saturated liquid
saturated vapor

Chart C-2SI

Piston Displacement

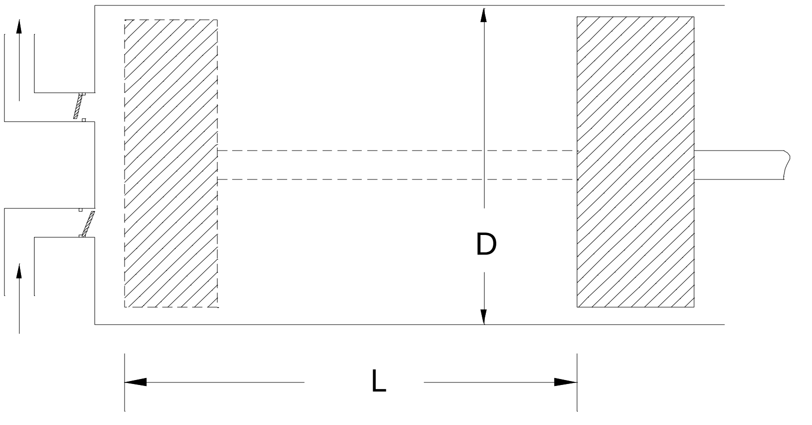
mass of the refrigerant in the cycle

$$\dot{m} = \rho \dot{V} = \frac{\dot{V}}{v} = \frac{PD_{th}}{v}$$

$$PD_{th} = v\dot{m} \;\; [\mathrm{m}^3 / s]$$

For a reciprocating compressor

with bore D, stroke L



$$PD_{th} = \frac{\pi D^2}{4} L \, n_p \, \frac{N}{60}$$