King Abdulaziz University
Mechanical Engineering Department

MEP365

Thermal Measurements

Signals & System Response

2022

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Signals & System Response (Ch.2 & Ch.3)

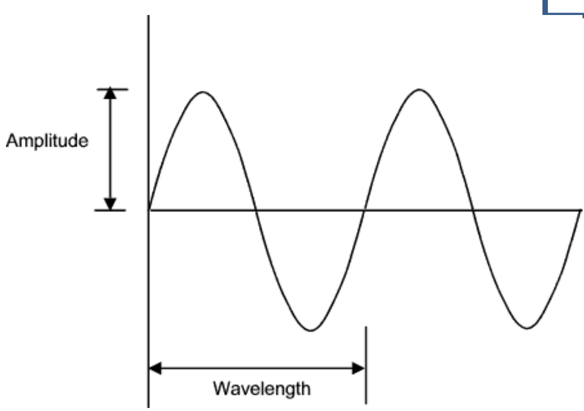
1-Measurment system (input signals and output signals)
2-Types of Signals
3-Average and RMS (Root-Mean-Squared) of a signal
4-Sinusoidal periodic signals
5-Fourier's wave form
6-Generlized form of differential equation for measuring system
7-Zero order system
8-First order system
    8a-step input and system response for first order system
    8b-frequency response for first order system
9-2nd order system
    9a-step response for 2nd order system
    9b-frequency response for 2nd order system

1-Measurment system (input signals and output signals)

Input signal

Measurement System
Differential Equation

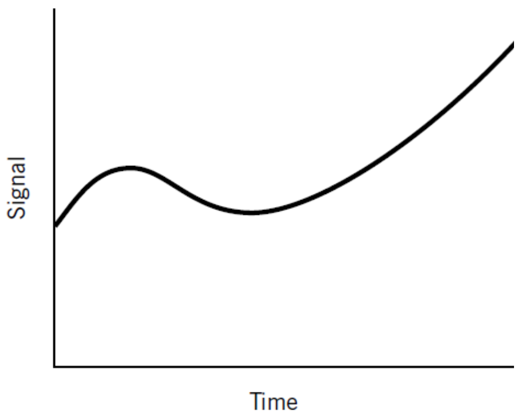
Output signal



Wave Signal
Amplitude and frequency

2-Types of signal

- Analog

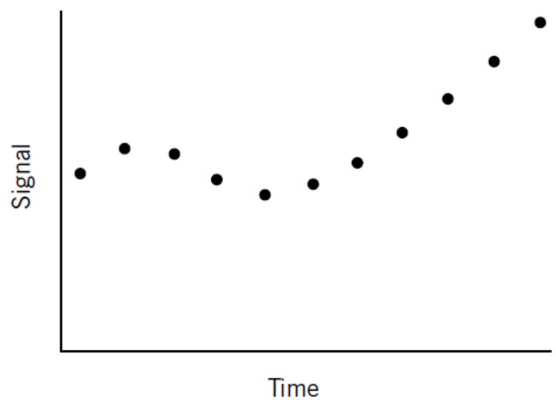


(a) Analog signal representation

Figure 2.2 Analog signal concepts.

Continuous variation with time

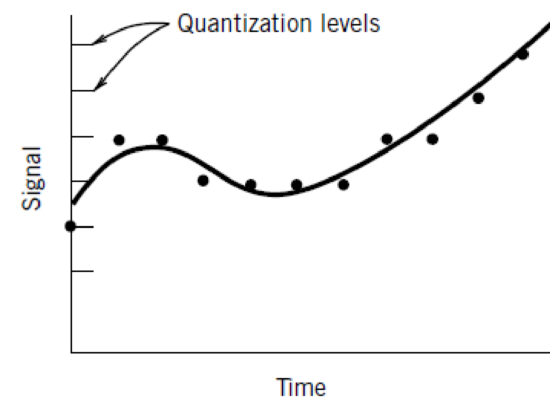
- Time-discrete



(a) Discrete time signal

Specified at certain interval of time

- Digital



(a) Digital signal

Figure 2.4 Digital signal representation

Specified at certain interval of time & certain levels

2-Types of signal

Static and Dynamic signals

Signals:

Static

Signal variation with time is negligible compared to time span of the measurements

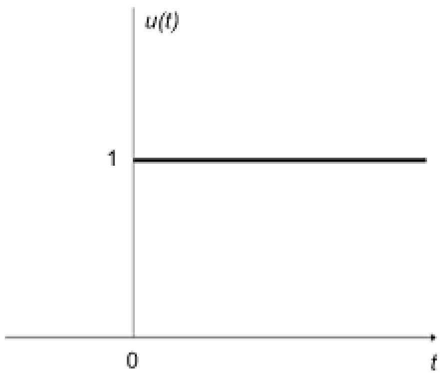
Example: measuring room temperature

Dynamic

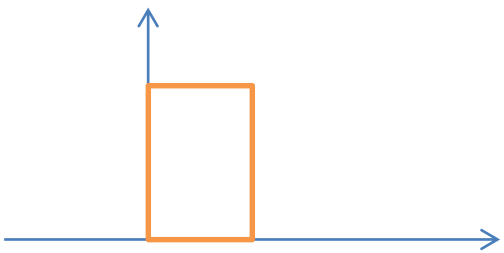
Signal variation with time is comparable to time span of the measurements

Example: measuring the pressure inside a cylinder of an internal combustion engine

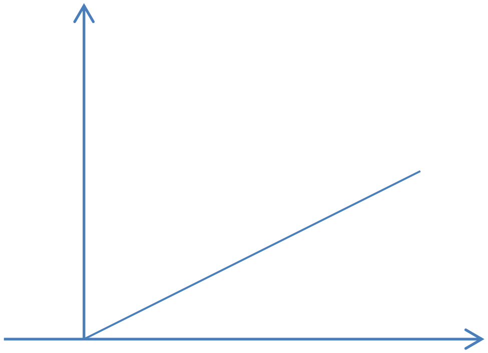
Samples of the input functions to measurement system



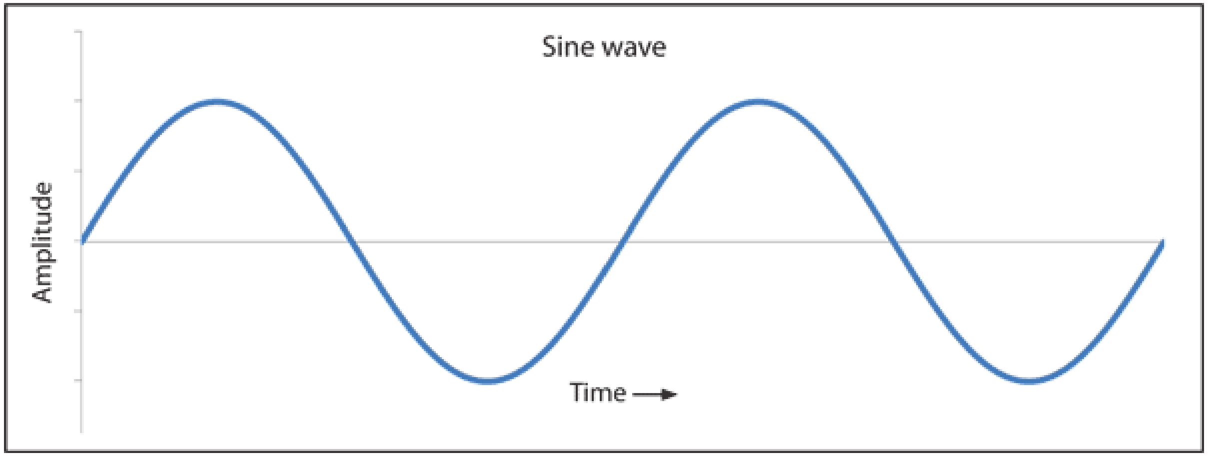
1-Step function



2-Impulse



3-Ramp



4-Sine wave

Samples of the input functions to measurement system

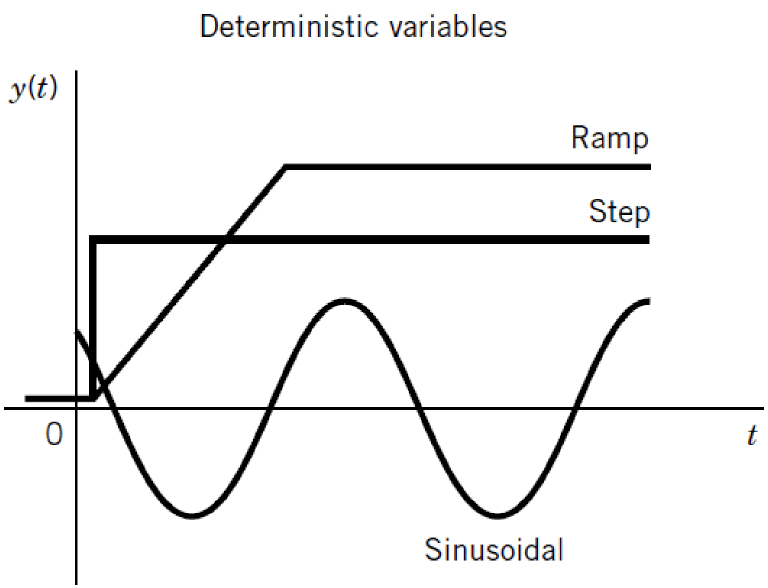
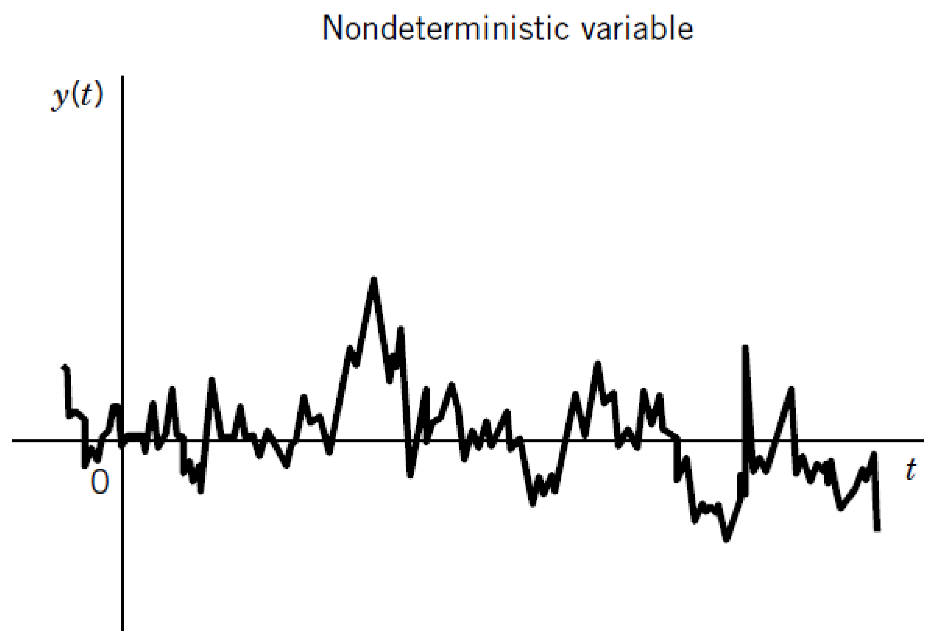


Figure 2.5 Examples of dynamic signals.



Samples of the input functions to measurement system

Table 2.1 Classification of Waveforms

I.	Static	$y(t) = A_0$
II.	Dynamic	
	Periodic waveforms	
	Simple periodic waveform	$y(t) = A_0 + C\sin(\omega t + \phi)$
	Complex periodic waveform	$y(t) = A_0 + \sum_{n=1}^{\infty} C_n \sin(n\omega t + \phi_n)$
	Aperiodic waveforms	
	Step[a]	$y(t) = A_0 U(t)$ $= A_0$ for $t > 0$
	Ramp	$y(t) = A_0 t$ for $0 < t < t_f$
	Pulse[b]	$y(t) = A_0 U(t) - A_0 U(t - t_1)$
III.	Nondeterministic waveform	$y(t) \approx A_0 + \sum_{n=1}^{\infty} C_n \sin(n\omega t + \phi_n)$

[a] $U(t)$ represents the unit step function, which is zero for $t < 0$ and 1 for $t \geq 0$.
[b] t_1 represents the pulse width.

Analog and digital representation of wave signal

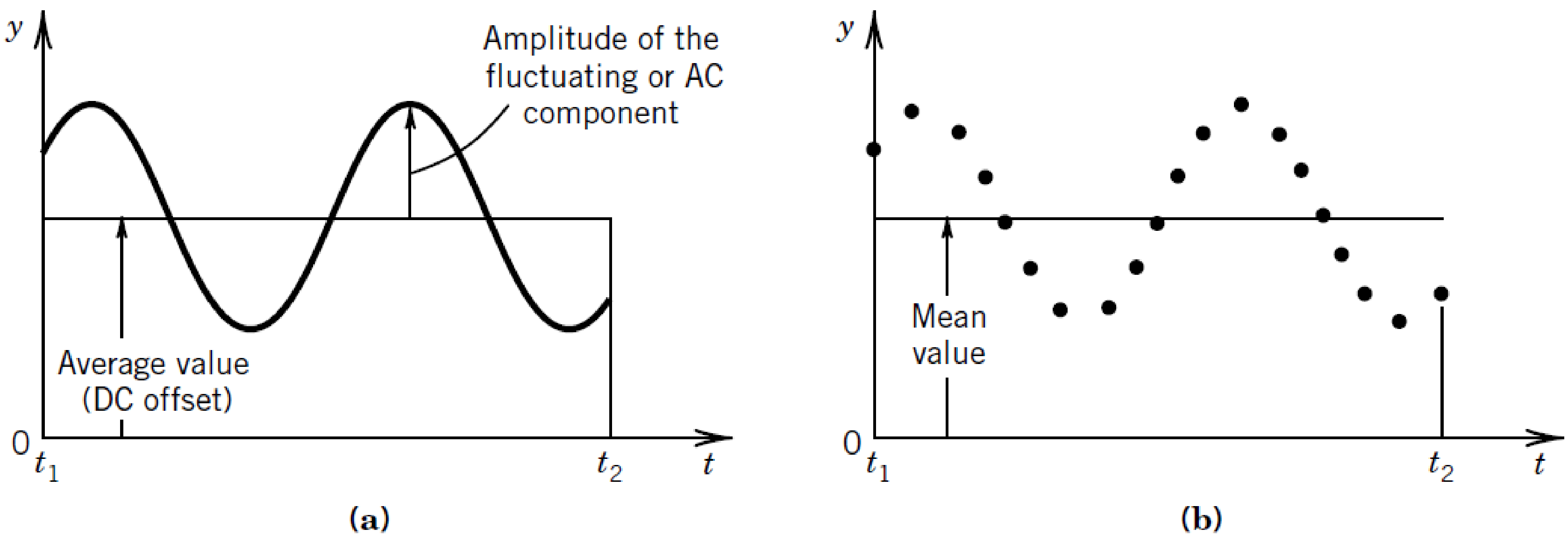


Figure 2.6 Analog and discrete representations of a dynamic signal.

3-Average and RMS (Root-Mean-Squared) of a signal

Function average

$$\bar{y} \equiv \frac{\int_{t_1}^{t_2} y(t)\,dt}{\int_{t_1}^{t_2} dt}$$

RMS=Root mean squared

$$y_{\mathrm{rms}} = \sqrt{\frac{1}{t_2 - t_1}\int_{t_1}^{t_2} y^2\,dt}$$

For discrete function

$$y(t) \to \{y(r\delta t)\} \quad r = 0, 1, \ldots, (N-1)$$

Average

$$\bar{y} = \frac{1}{N}\sum_{i=0}^{N-1} y_i$$

RMS value

$$y_{\mathrm{rms}} = \sqrt{\frac{1}{N}\sum_{i=0}^{N-1} y_i^2}$$

Subtracting the DC offset from the signal

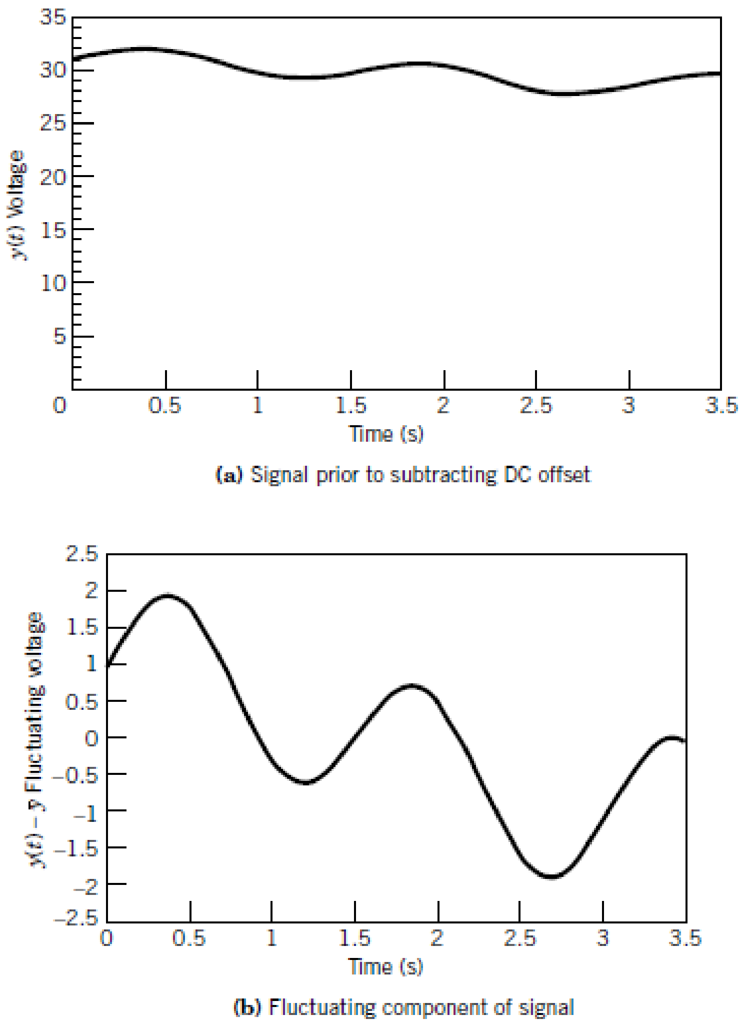


Figure 2.7 Effect of subtracting DC offset for a dynamic signal.

Example 2.1

Evaluation of mean and RMS of a periodic wave

Suppose the current passing through a resistor can be described by

$$I(t) = 10 \sin t$$

KNOWN $I(t) = 10 \sin t$

FIND $\bar{I}$ and I_{rms} with $t_f = \pi$ and 2π

SOLUTION The average value for a time from 0 to t_f is found from Equation 2.1 as

$$\bar{I} = \frac{\int_0^{t_f} I(t)dt}{\int_0^{t_f} dt} = \frac{\int_0^{t_f} 10 \sin t dt}{t_f}$$

Evaluation of this integral yields

$$\bar{I} = \frac{1}{t_f}[-10 \cos t]_0^{t_f}$$

With $t_f = \pi$, the average value, $\bar{I}$, is $20/\pi$. For $t_f = 2\pi$, the evaluation of the integral yields an average value of zero.

The rms value for the time period 0 to t_f is given by the application of Equation 2.4, which yields

$$I_{\text{rms}} = \sqrt{\frac{1}{t_f}\int_0^{t_f} I(t)^2 dt} = \sqrt{\frac{1}{t_f}\int_0^{t_f} (10 \sin t)^2 dt}$$

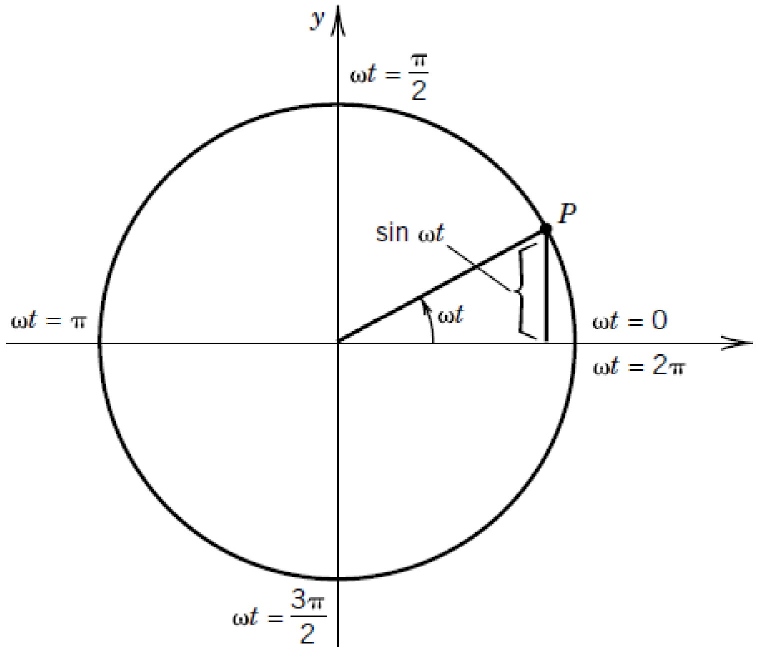
This integral is evaluated as

$$I_{\text{rms}} = \sqrt{\frac{100}{t_f}\left(-\frac{1}{2}\cos t \sin t + \frac{t}{2}\right)\Big|_0^{t_f}}$$

For $t_f = \pi$, the rms value is $\sqrt{50}$. Evaluation of the integral for the rms value with $t_f = 2\pi$ also yields $\sqrt{50}$.

4-Sinusoidal periodic signals

Signal amplitude and frequency

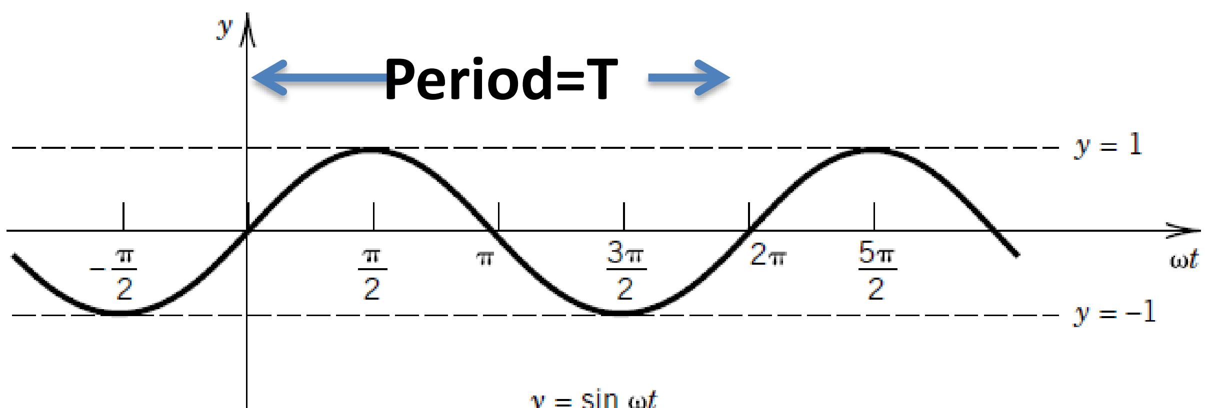


$y = \sin(\omega t)$

ω Circular frequency [rad/s]

T=period [s]

f=frequency [1/s]=Hz
f=1/T



$$T = \frac{2\pi}{\omega} = \frac{1}{f}$$

$$\omega = 2\pi f$$

Simple mass spring system

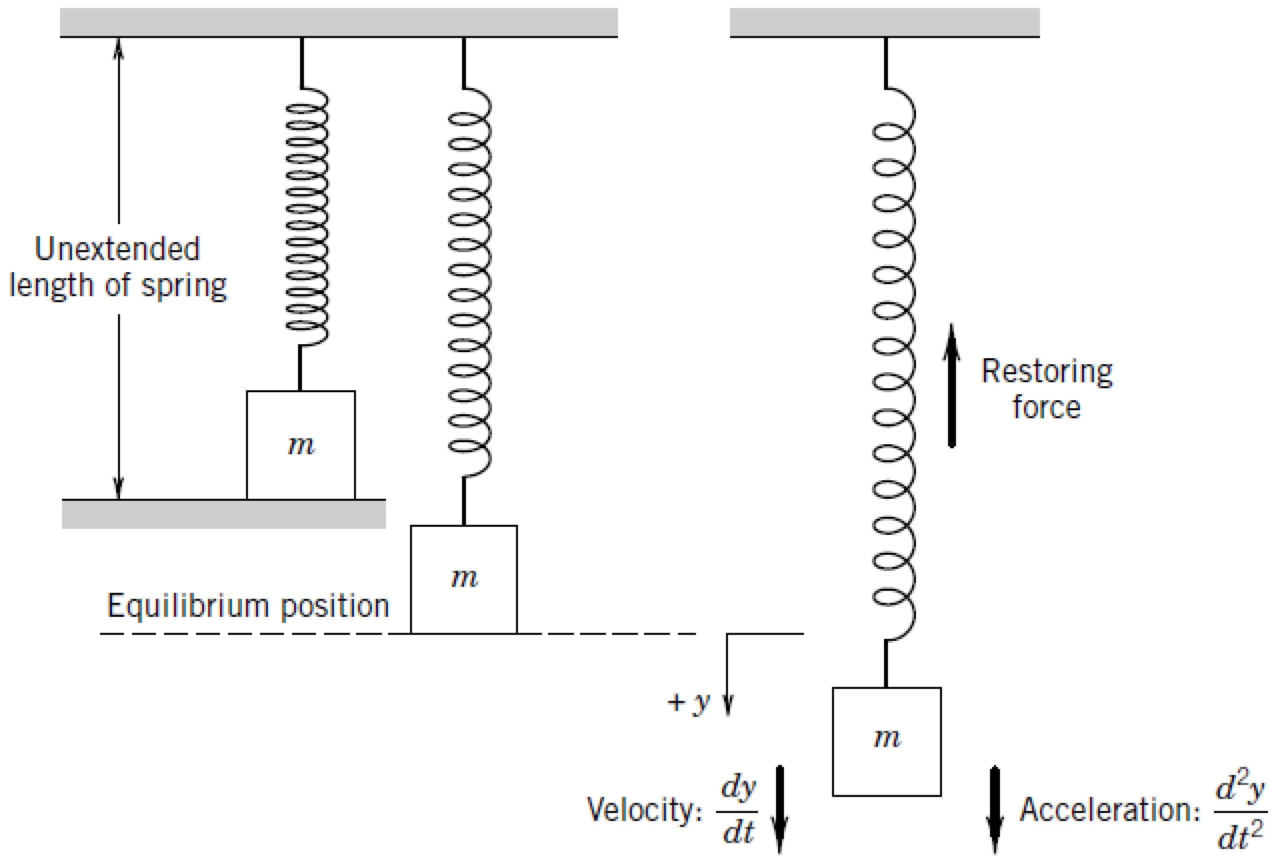


Figure 2.9 Spring-mass system.

Applying force balance

$$\sum F = ma$$

For free body:

$$m\,\frac{d^2y}{dt^2} + ky = 0$$

Spring force

$$F = ky$$

Simple mass spring system

Differential equation

$$m\frac{d^2y}{dt^2} + ky = 0$$

Solution form

$$y = A\cos\omega t + B\sin\omega t$$

Can be put in form of only sin or cos functions such as

$$y = C\cos(\omega t - \phi)$$

$$y = C\sin(\omega t + \phi^*)$$

Useful relations for sin & cos functions

$$A\cos\omega t + B\sin\omega t = \sqrt{A^2+B^2}\cos(\omega t - \phi)$$

$$A\cos\omega t + B\sin\omega t = \sqrt{A^2+B^2}\sin(\omega t + \phi^*)$$

$$\phi = \tan^{-1}\frac{B}{A} \quad \phi^* = \tan^{-1}\frac{A}{B} \quad \phi^* = \frac{\pi}{2} - \phi$$

$$y = C\cos(\omega t - \phi)$$

$$y = C\sin(\omega t + \phi^*)$$

5-Fourier's wave form

Any function with period T=2π can be written as combination of sin and cos function (i.e. Fourier's form)

$$y(t) = A_0 + \sum_{n=1}^{\infty} (A_n \cos nt + B_n \sin nt)$$

$$A_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} y(t)$$

$$A_n = \frac{1}{\pi} \int_{-\pi}^{\pi} y(t) \cos nt\, dt$$

$$B_n = \frac{1}{\pi} \int_{-\pi}^{\pi} y(t) \sin nt\, dt$$

5-Fourier's wave form

A function with period T can be also be presented by

$$A_0 = \frac{1}{T}\int_{-T/2}^{T/2} y(t)dt$$

$$A_n = \frac{2}{T}\int_{-T/2}^{T/2} y(t)\cos n\omega t dt$$

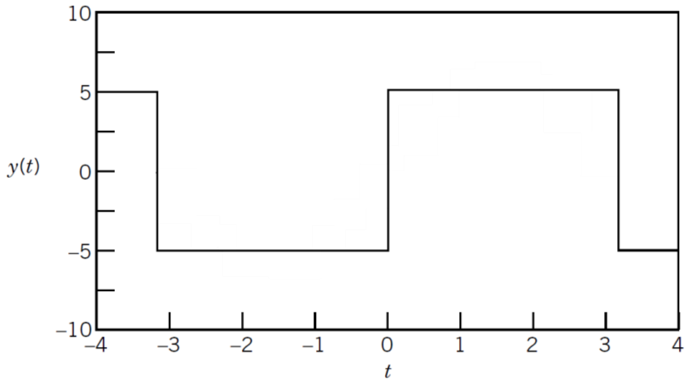
$$B_n = \frac{2}{T}\int_{-T/2}^{T/2} y(t)\sin n\omega t dt$$

$$y(t) = A_0 + \sum_{n=1}^{\infty} (A_n \cos n\omega t + B_n \sin n\omega t)$$

5-Fourier's wave form

Example 2.4 :

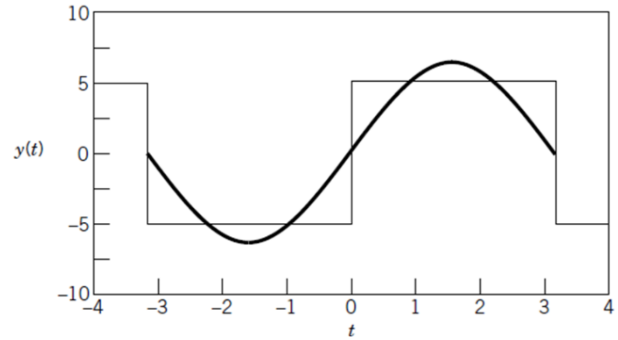
Find the Fourier coefficient for a square periodic function as shown below



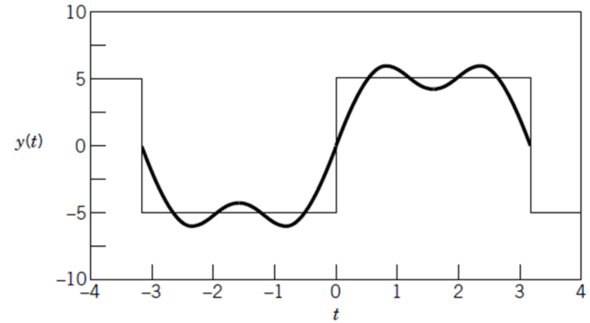
Representation of a square wave using Fourier's functions

$$y(t) = \frac{20}{\pi}\left(\sin t + \frac{1}{3}\sin 3t + \frac{1}{5}\sin 5t + \cdots\right)$$

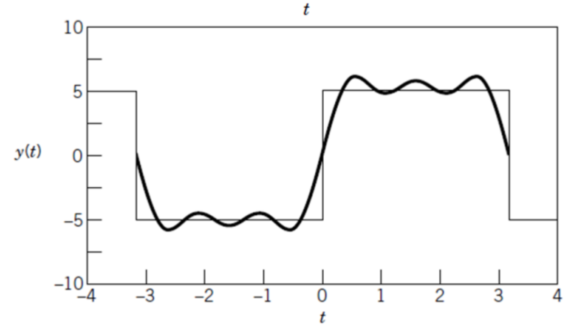
First partial sum



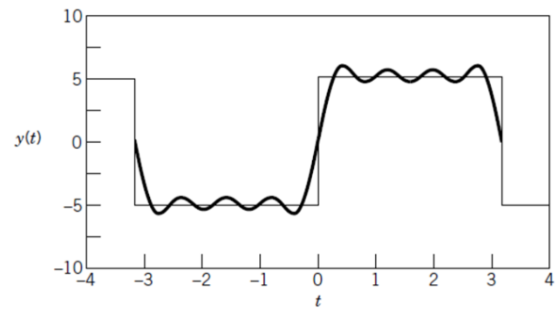
Second partial sum



Third partial sum



Fourth partial sum



Chapter 3

Measurement System Behavior

Interaction of a measurement system with input signal

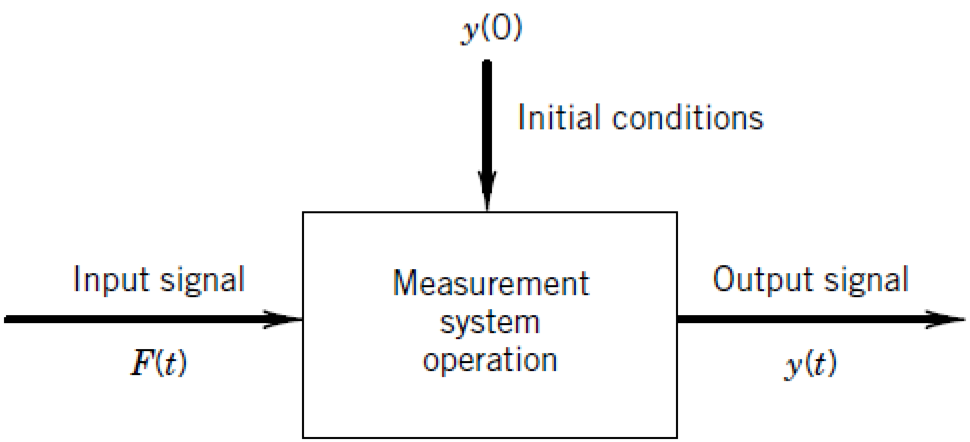


Figure 3.2 Measurement system operation on an input signal, $F(t)$, provides the output signal, $y(t)$.

6- General form of the differential equation of a measuring system

$$a_n \frac{d^n y}{dt^n} + a_{n-1} \frac{d^{n-1} y}{dt^{n-1}} + \cdots + a_1 \frac{dy}{dt} + a_0 y = F(t)$$

n is the order of the differential equation

7- Zero-order systems

Differential equation $a_0 y = F(t)$

The solution

$$y(t) = KF(t)$$

$$K = \frac{1}{a_0}$$

K = static sensitivity

8- First order system

Differential equation form

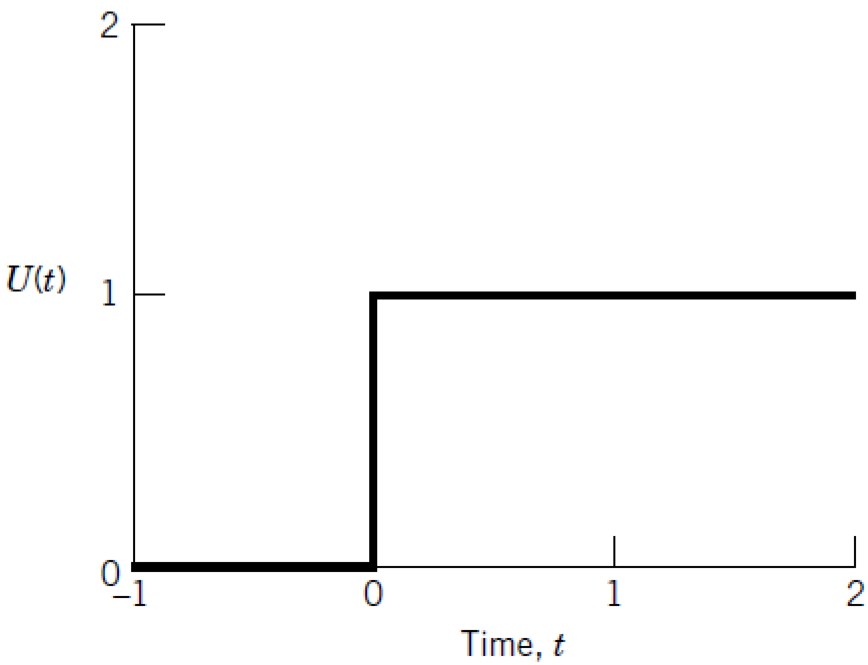
$$a_1 \frac{dy}{dt} + a_0 y = F(t)$$

$$\frac{a_1}{a_0}\frac{dy}{dt} + y = \frac{F(t)}{a_0}$$

$$\tau \frac{dy}{dt} + y = K\mathrm{F(t)}$$

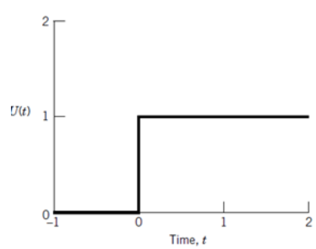
τ is the time constant [s]

8a-step input and system response for first order system



$$F(t) = AU(t)$$

Figure 3.5 The unit step function, $U(t)$.



Step input

$$\tau \frac{dy}{dt} + y = KF(t)$$

First order measurement system

Output Signal

8a-step input and system response for first order system

Solution for first order system exposed to step input

$$\tau\dot{y} + y = KF(t) = KAU(t)$$

Solution y consists of two parts; homogenous solution and particular solution

$$y(t) = y_p + y_h$$

$$\underbrace{y(t)}_{\text{Time response}} = \underbrace{KA}_{\text{Steady response}} + \underbrace{(y_0 - KA)e^{-t/\tau}}_{\text{Transient response}}$$

$$y - y_0 = (KA - y_0)(1 - e^{-t/\tau})$$

Subtract y_∞ form both sides to get

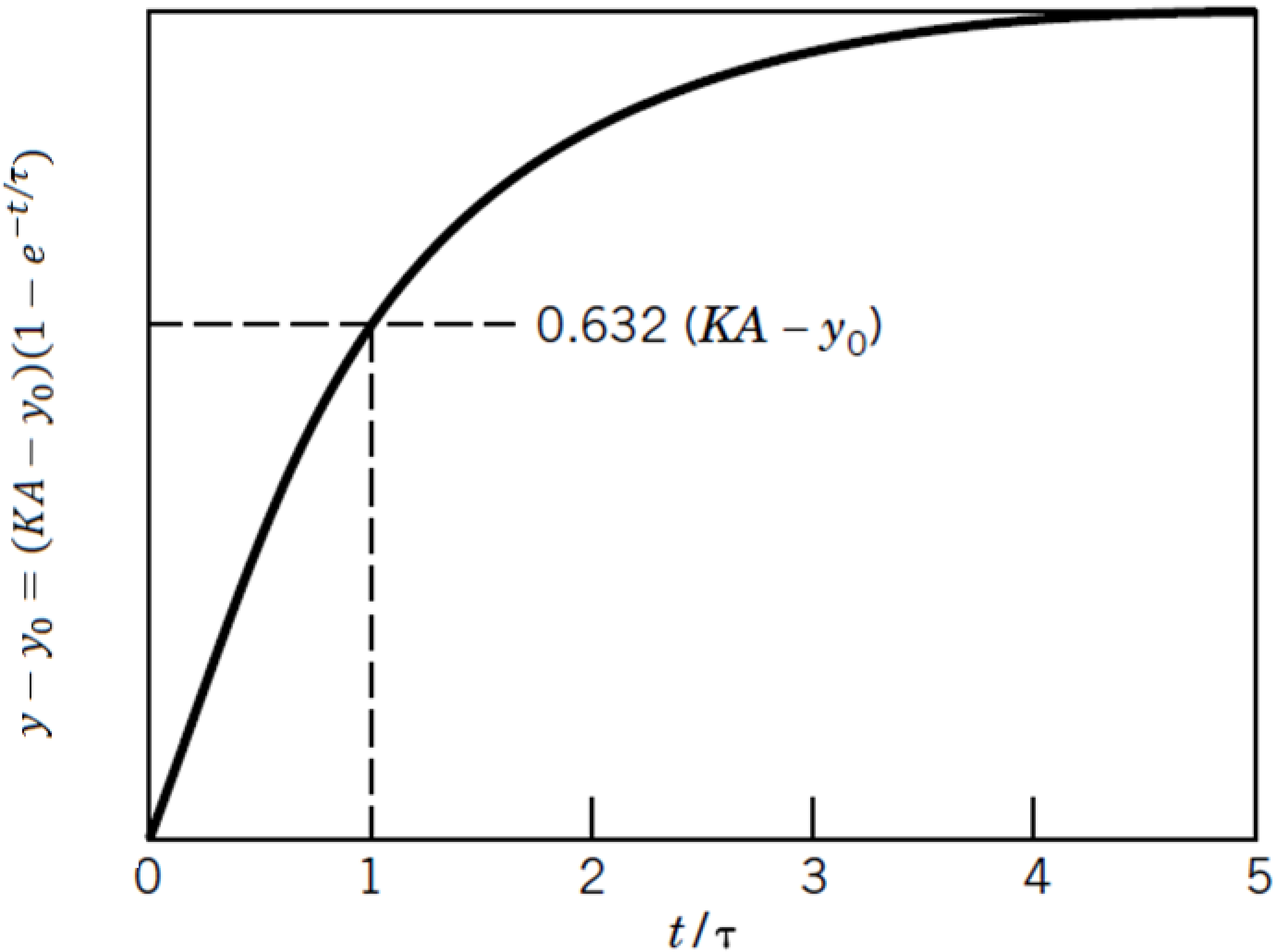
Error fraction function

$$\Gamma(t) = \frac{y(t) - y_\infty}{y_0 - y_\infty} = e^{-t/\tau}$$

Here

$$y_\infty = KA$$

Output of first order system exposed to step input KA with initial $y=y_0$



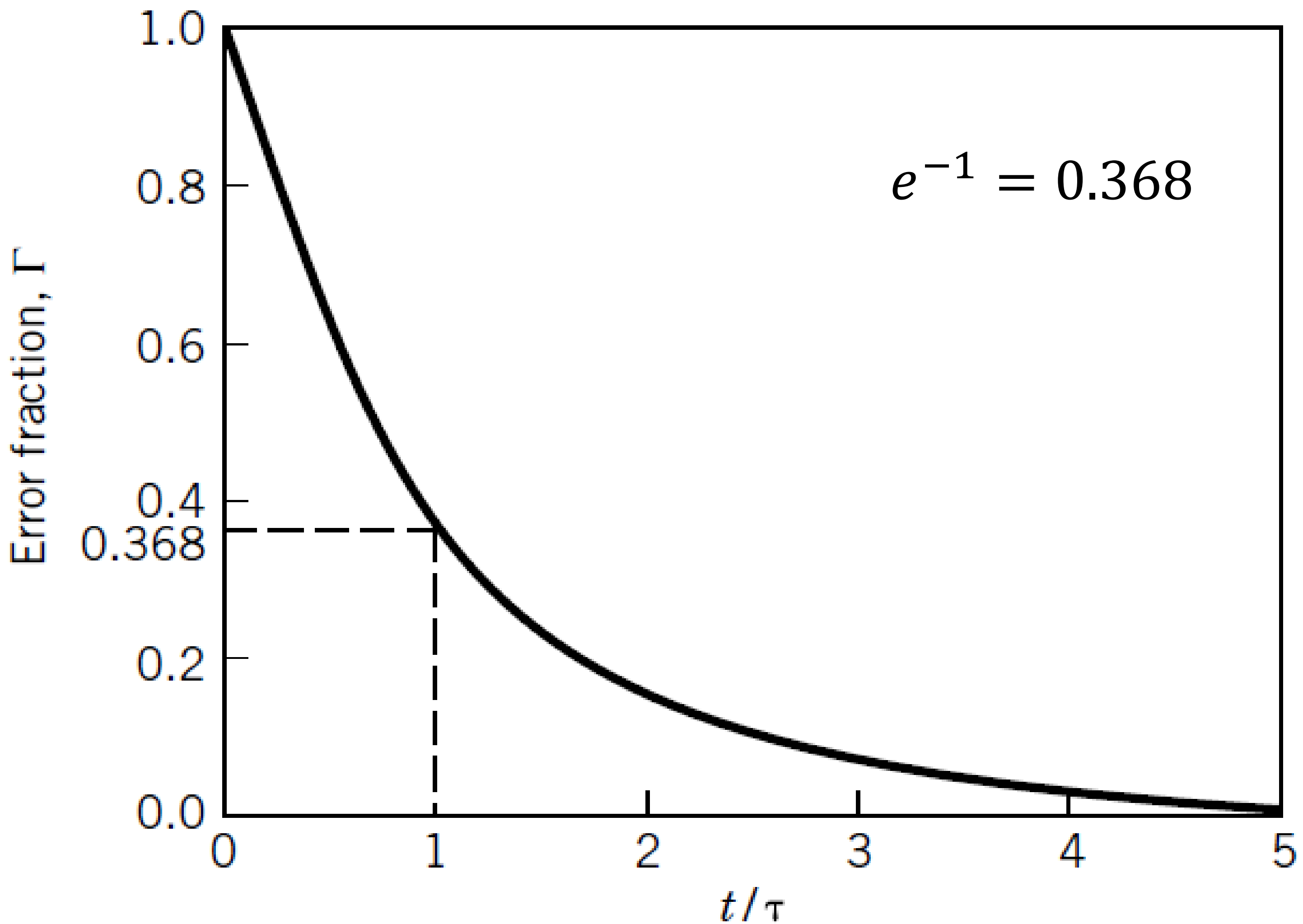
$$y_\infty = KA$$

$$e^{-1} = 0.368$$

Solution for first order system exposed to step input

Error Fraction function

$$\Gamma(t) = \frac{y(t) - y_\infty}{y_0 - y_\infty} = e^{-t/\tau}$$



Shape of the error fraction plotted on semilog coordinates

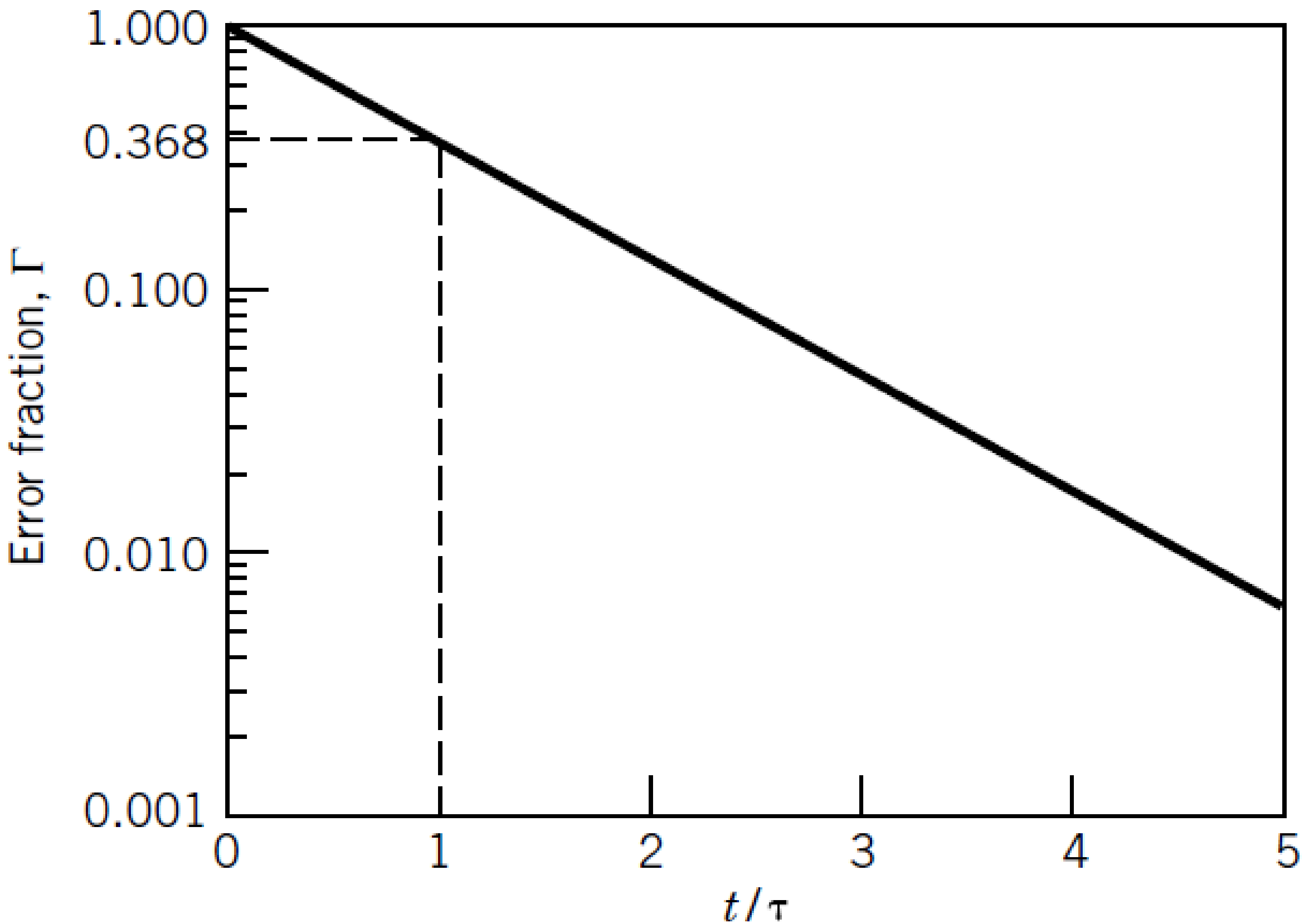


Figure 3.8 The error fraction plotted on semilog coordinates.

Experimentally finding the time constant for a first order system

- Plotting on semilog plot the variation in gamma Γ (error function) vs time to get accurate value of the time constant τ
- Use curvefit to find the best equation fit
- From the figure estimate the slope of the curve which is related to the time constant

$$\Gamma(t) = \frac{y(t) - y_\infty}{y_0 - y_\infty} = e^{-t/\tau}$$

$$\log(\Gamma) = -t/\tau$$

$$Y = mt$$

See Example 3.5 in your textbook

$m = -1/\tau$

Example 3.5

A particular thermometer is subjected to a step change, such as in Example 3.3, in an experimental exercise to determine its time constant. The temperature data are recorded with time and presented in Figure 3.10. Determine the time constant for this thermometer. In the experiment, the heat transfer coefficient, h, is estimated to be 6 W/m^2-°C from engineering handbook correlations.

KNOWN Data of Figure 3.10

$h = 6\ \mathrm{W/m^2\text{-}°C}$

ASSUMPTIONS First-order behavior using the model of Example 3.3, constant properties

FIND τ

SOLUTION According to Equation 3.7, the time constant should be the negative reciprocal of the slope of a line drawn through the data of Figure 3.10. Aside from the first few data points, the data appear to follow a linear trend, indicating a nearly first-order behavior and validating our model assumption. The data is fit to the first-order equation[2]

$$2.3 \log \Gamma = (-0.194)t + 0.00064$$

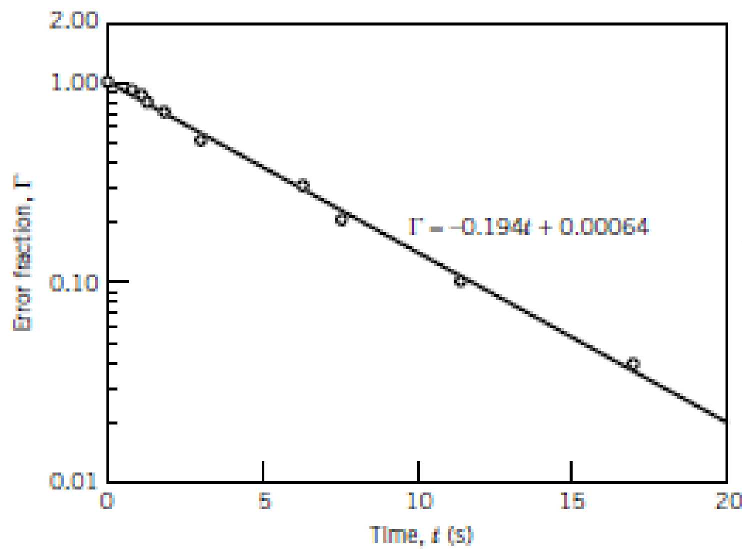


Figure 3.10 Temperature–time history of Example 3.5.

$$\Gamma(t) = \frac{y(t) - y_\infty}{y_0 - y_\infty} = e^{-t/\tau}$$

$$slope = m = -0.194 = -1/\tau$$

$$\tau = 5.15\ s$$

[2] The least-squares approach to curve fitting is discussed in detail in Chapter 4.

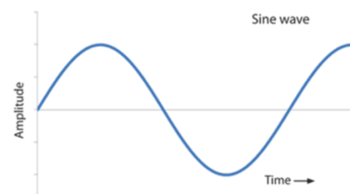
8b-frequency response for first order system

Assume the input function to be

$$F(t) = A\sin(\omega t)$$

The first order differential equation of the measuring system is

$$\tau\dot{y} + y = KA\sin\omega t$$



$$\tau\frac{dy}{dt} + y = K\mathrm{F(t)}$$

Output signal

First order measurement system

The solution of the differential equation

$$y(t) = Ce^{-t/\tau} + \frac{KA}{\sqrt{1+(\omega\tau)^2}}\sin\left(\omega t - \tan^{-1}\omega\tau\right)$$

Which consists of a decayed part plus a sinusoidal part

After certain time the decayed part will disappear and the left is the sinusoidal part

8b-frequency response for first order system

The solution of the differential equation

$$y(t) = Ce^{-t/\tau} + B(\omega) sin[\omega t + \Phi]$$

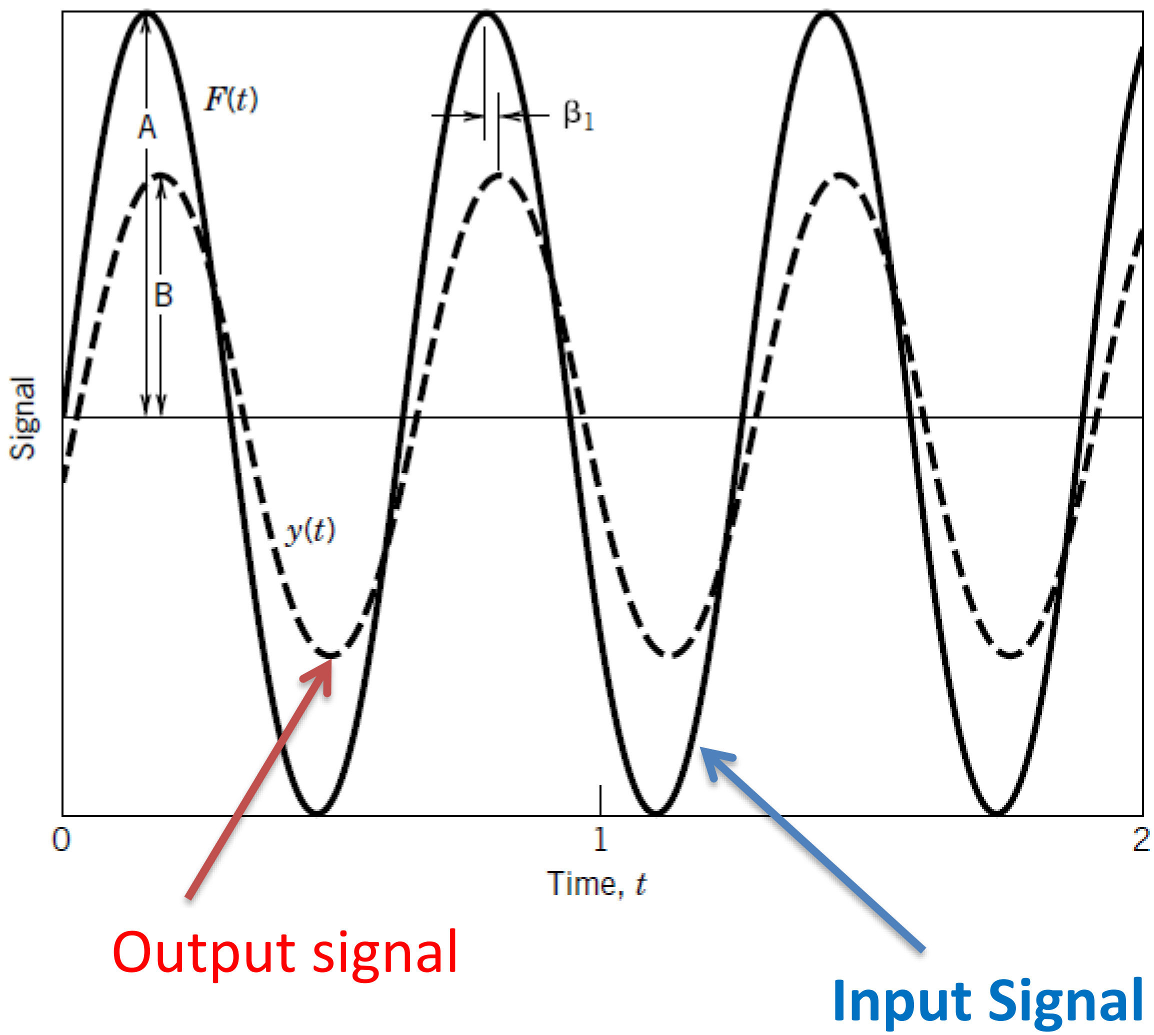
$$B(\omega) = \frac{KA}{\sqrt{1+(\omega\tau)^2}}$$

Magnitude $$M(\omega) = \frac{B}{KA} = \frac{1}{\sqrt{1+(\omega\tau)^2}}$$

Phase shift $$\Phi(\omega) = -\tan^{-1}(\omega\tau)$$

Φ Is the phase shift

Shape of the input and output signals on first order measuring system



Time delay

$$\beta_1 = \frac{\Phi}{\omega}$$

Figure 3.11 Relationship between a sinusoidal input and output: amplitude, frequency, and time delay.

8b-frequency response for first order system

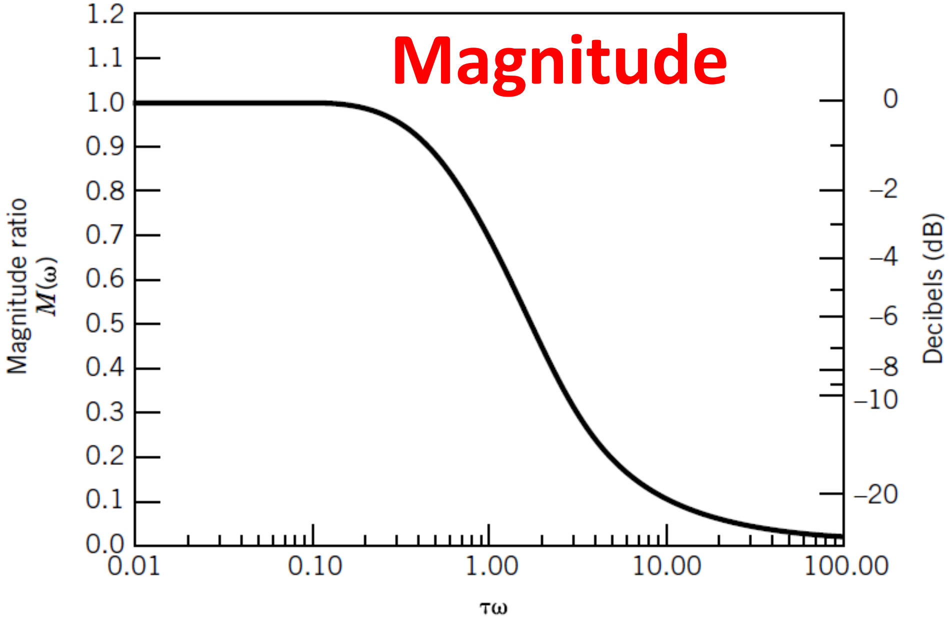
Define the function M(ω) which is the magnitude ratio

$$M(\omega) = \frac{B}{KA} = \frac{1}{\sqrt{1 + (\omega\tau)^2}}$$

When $\omega \rightarrow 0$ $M(\omega) \rightarrow 1$

When $\omega \rightarrow \infty$ $M(\omega) \rightarrow 0$

When $\omega \rightarrow 1/\tau$ $M(\omega) \rightarrow 0.707$ or -3dB



Dynamic error=δ

$$\delta(\omega) = M(\omega) - 1$$

Decibels

$$\mathrm{dB} = 20 \log M(\omega)$$

8b-frequency response for first order system

Phase shift

$$\Phi = -\tan^{-1}(\omega\tau)$$

When $\omega \to 0$ $\quad \Phi(\omega) \to 0$

When $\omega \to \infty$ $\quad \Phi(\omega) \to -90$

When $\omega \to 1/\tau$ $\quad \Phi(\omega) \to -45$

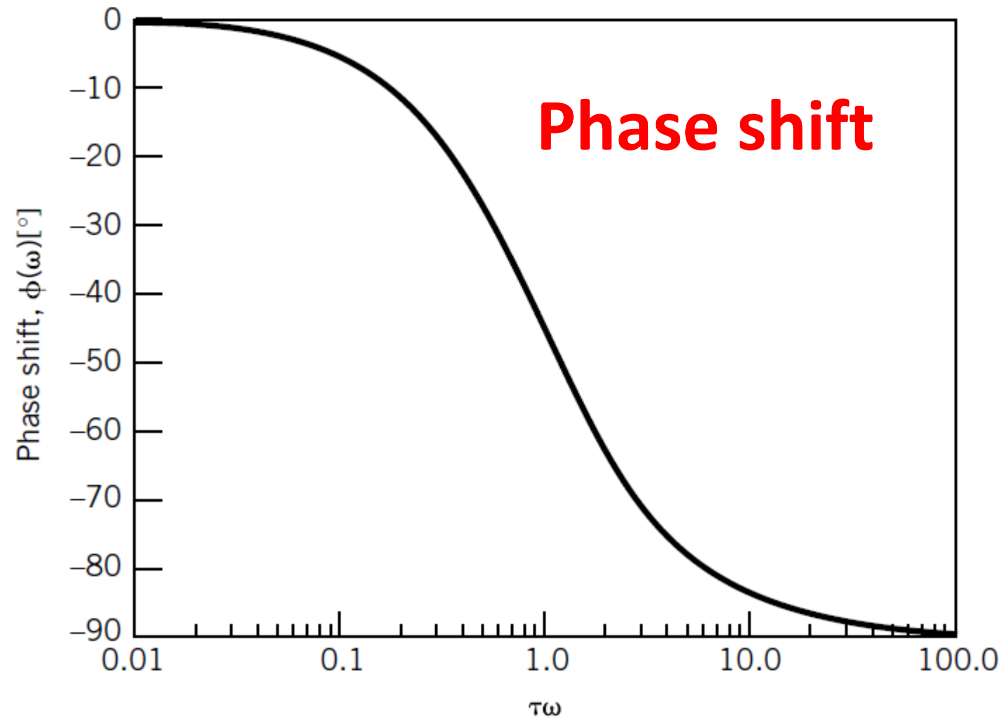
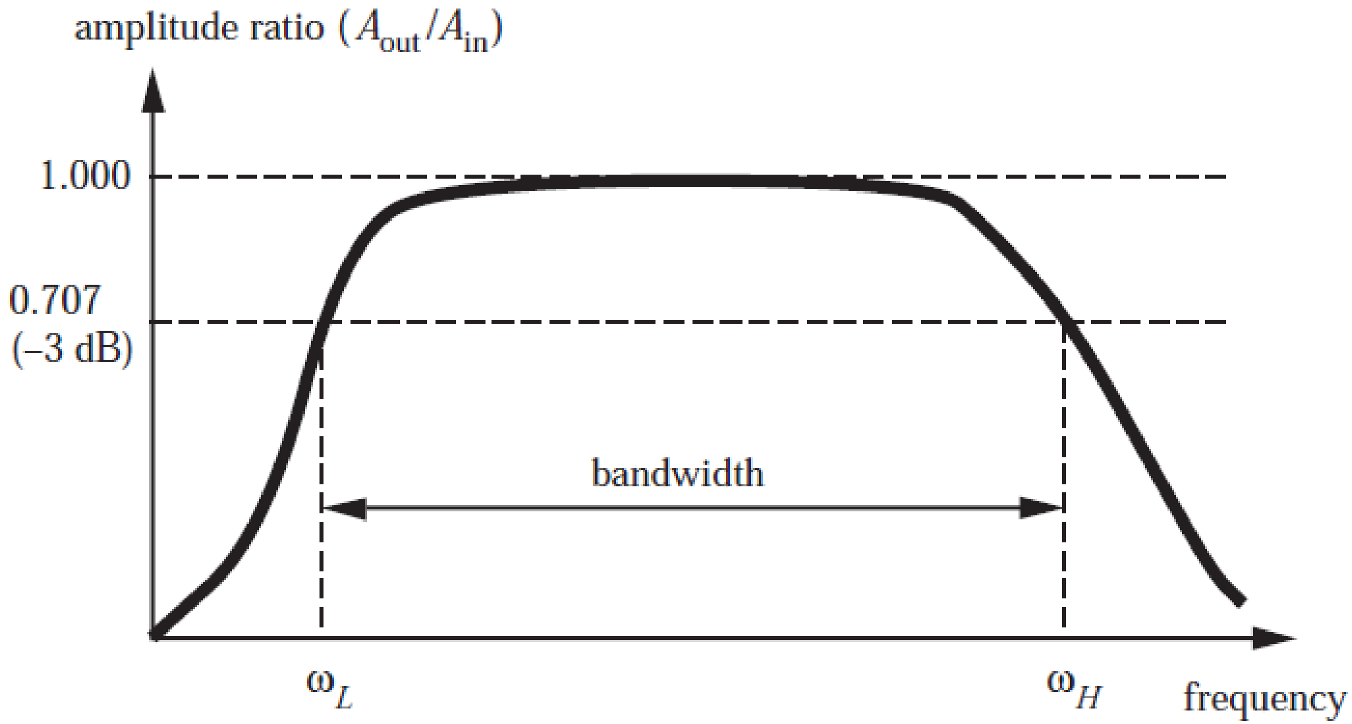


Figure 3.13 First-order system frequency response: phase shift.

Definition of bandwidth

$$dB = 20logM(\omega)$$



Frequency bandwidth= the frequency band over which M(ω) <= 0.707. In terms of decibels, the band frequencies within which M(ω) remains above -3dB

9-2nd order system

$$a_2\ddot{y} + a_1\dot{y} + a_0 y = F(t)$$

$$m\frac{d^2y}{dt^2} + c\frac{dy}{dt} + ky = F(t)$$

Input signal

$$\boxed{a_2\ddot{y} + a_1\dot{y} + a_0 y = F(t)}$$

Output Signal

Second order measurement system

$$\frac{1}{\omega_n^2}\ddot{y} + \frac{2\zeta}{\omega_n}\dot{y} + y = KF(t)$$

$$\omega_n = \sqrt{\frac{a_0}{a_2}} = \text{natural frequency of the system}$$

$$\zeta = \frac{a_1}{2\sqrt{a_0 a_2}} = \text{damping ratio of the system}$$

Examples of 2nd order systems:

- Accelerometer
- Pressure transducer

Damping coefficient $c_c = \frac{1}{(2\sqrt{km})}$

$$\omega_n = \sqrt{\frac{k}{m}} \quad \zeta = \frac{c}{2\sqrt{km}}$$

9-2nd order system

$$\frac{1}{\omega_n^2}\lambda^2 + \frac{2\zeta}{\omega_n}\lambda + 1 = 0$$

The roots of the characteristic function (for the homogeneous equation)

$$\lambda_{1,2} = -\zeta\omega_n \pm \omega_n\sqrt{\zeta^2 - 1}$$

Roots can be either

1-Real & un -equal two roots

2-Two equal roots

3-Complex roots

9-2nd order system

Homogenous solution

Depending on the value for ζ three forms of homogeneous solution are possible:

$0 \le \zeta < 1$ (underdamped system solution)

$$y_h(t) = Ce^{-\zeta\omega_n t}\sin\left(\omega_n\sqrt{1-\zeta^2}t + \Theta\right)$$

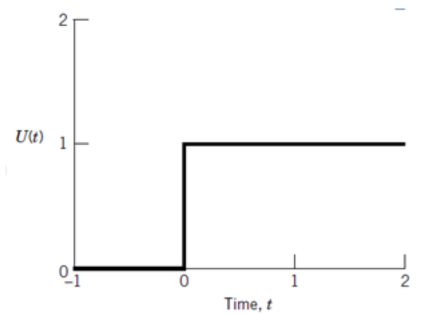
$\zeta = 1$ (critically damped system solution)

$$y_h(t) = C_1 e^{\lambda_1 t} + C_2 t e^{\lambda_2 t}$$

$\zeta > 1$ (overdamped system solution)

$$y_h(t) = C_1 e^{\lambda_1 t} + C_2 e^{\lambda_2 t}$$

9a-Rsponse of 2nd order system due to step input



Step input

Output signal

$$a_2\ddot{y} + a_1\dot{y} + a_0 y = F(t)$$

Second order measurement system

Response due to step input function

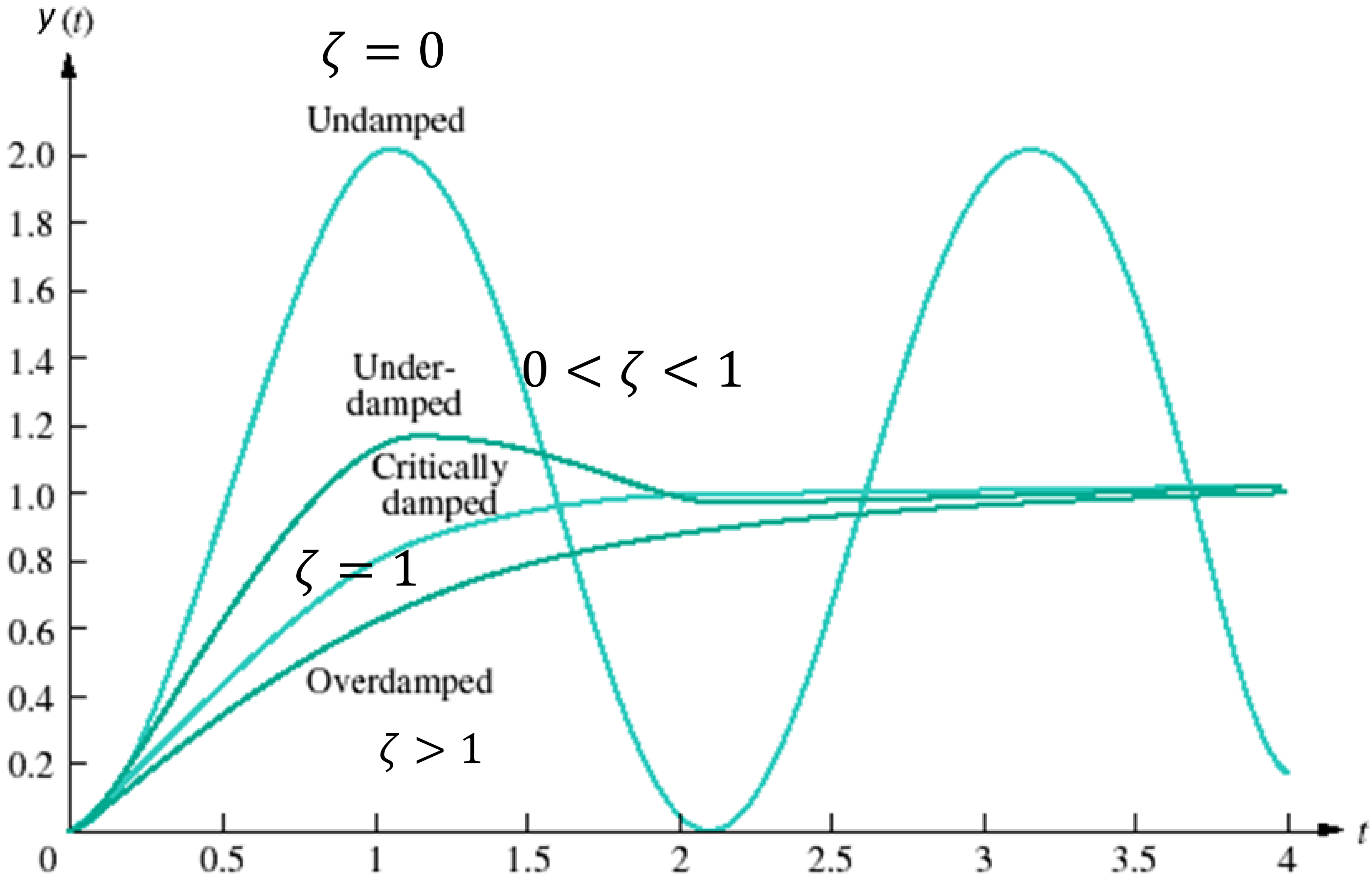
$$y(t) = KA - KAe^{-\zeta\omega_n t}\left[\frac{\zeta}{\sqrt{1-\zeta^2}}\sin\left(\omega_n t\sqrt{1-\zeta^2}\right) + \cos\left(\omega_n t\sqrt{1-\zeta^2}\right)\right] \quad 0 \le \zeta < 1 \quad (3.15a)$$

$$y(t) = KA - KA(1+\omega_n t)e^{-\omega_n t} \quad \zeta = 1 \quad (3.15b)$$

$$y(t) = KA - KA\left[\frac{\zeta+\sqrt{\zeta^2-1}}{2\sqrt{\zeta^2-1}}e^{\left(-\zeta+\sqrt{\zeta^2-1}\right)\omega_n t} + \frac{\zeta-\sqrt{\zeta^2-1}}{2\sqrt{\zeta^2-1}}e^{\left(-\zeta-\sqrt{\zeta^2-1}\right)\omega_n t}\right] \quad \zeta > 1 \quad (3.15c)$$

the initial conditions, $y(0) = \dot{y}(0) = 0$ for convenience.

Response due to step input function



Response due to step input function

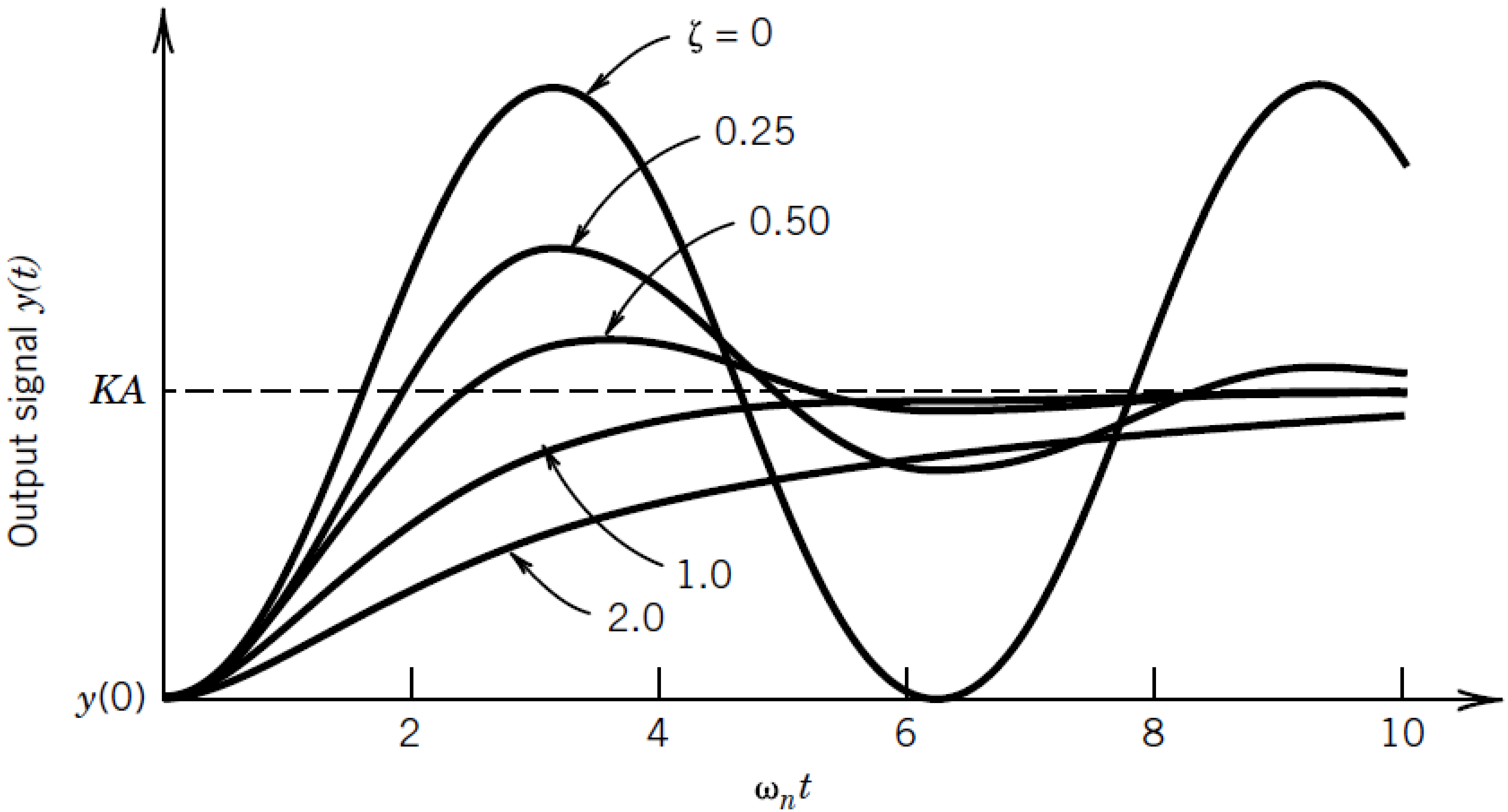


Figure 3.14 Second-order system time response to a step function input.

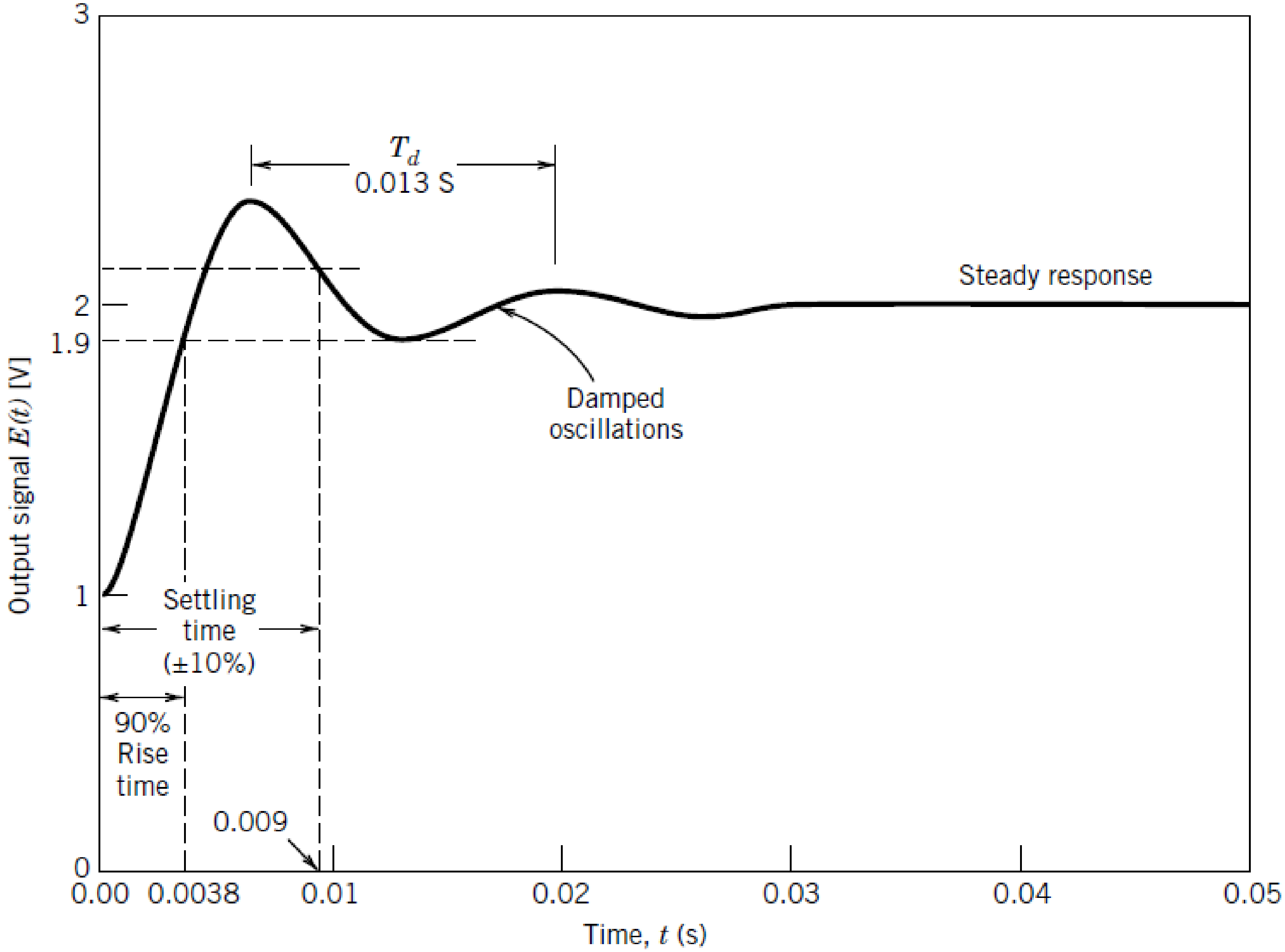
Some definitions (underdamped system)

Period

$$T_d = \frac{2\pi}{\omega_d} = \frac{1}{f_d}$$

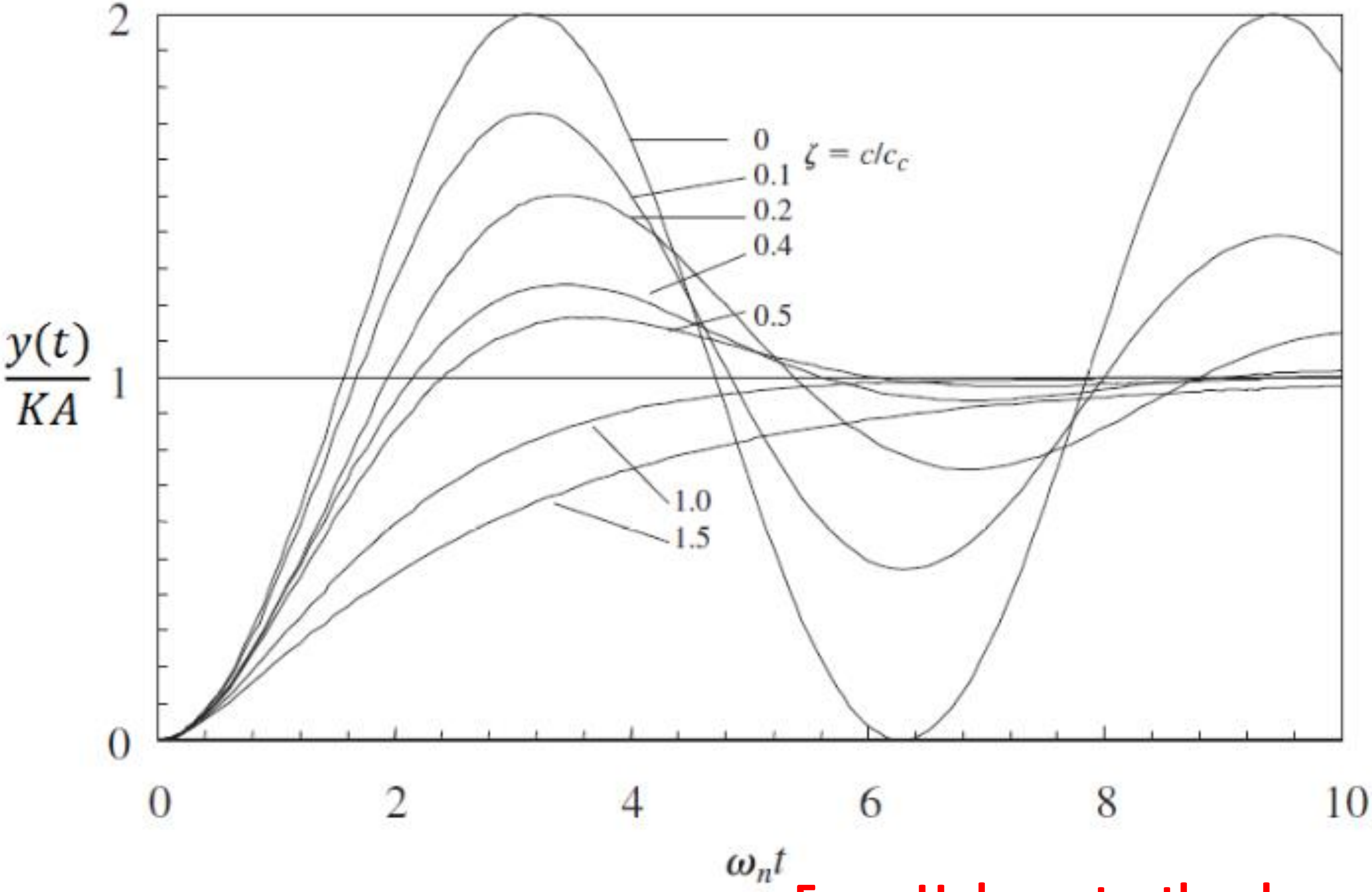
Ringing frequency

$$\omega_d = \omega_n\sqrt{1-\zeta^2}$$



where ω_d is called the *ringing frequency*.

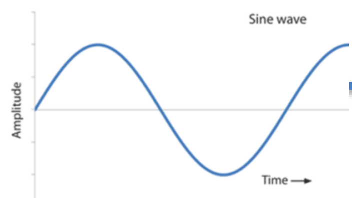
Response due to step input function



From Holman textbook

Response of 2nd order system due to sinusoidal input function

Output signal



$$a_2\ddot{y} + a_1\dot{y} + a_0 y = F(t)$$

???

Second order measurement system

Sinusoidal input

9b-frequency response

$$y(t) = y_h + \frac{KA\sin[\omega t + \Phi(\omega)]}{\left\{\left[1 - (\omega/\omega_n)^2\right]^2 + [2\zeta\omega/\omega_n]^2\right\}^{1/2}}$$

$$y_{\text{steady}}(t) = y(t \to \infty) = B(\omega)\sin[\omega t + \Phi(\omega)]$$

Or the magnitude will be

$$M(\omega) = \frac{B(\omega)}{KA} = \frac{1}{\left\{\left[1 - (\omega/\omega_n)^2\right]^2 + [2\zeta\omega/\omega_n]^2\right\}^{1/2}}$$

9b-frequency response

Magnitude

$$M(\omega) = \frac{B(\omega)}{KA} = \frac{1}{\left\{\left[1-(\omega/\omega_n)^2\right]^2 + [2\zeta\omega/\omega_n]^2\right\}^{1/2}}$$

Phase shift

$$\Phi(\omega) = \tan^{-1}\left(-\frac{2\zeta\omega/\omega_n}{1-(\omega/\omega_n)^2}\right)$$

9b-frequency response

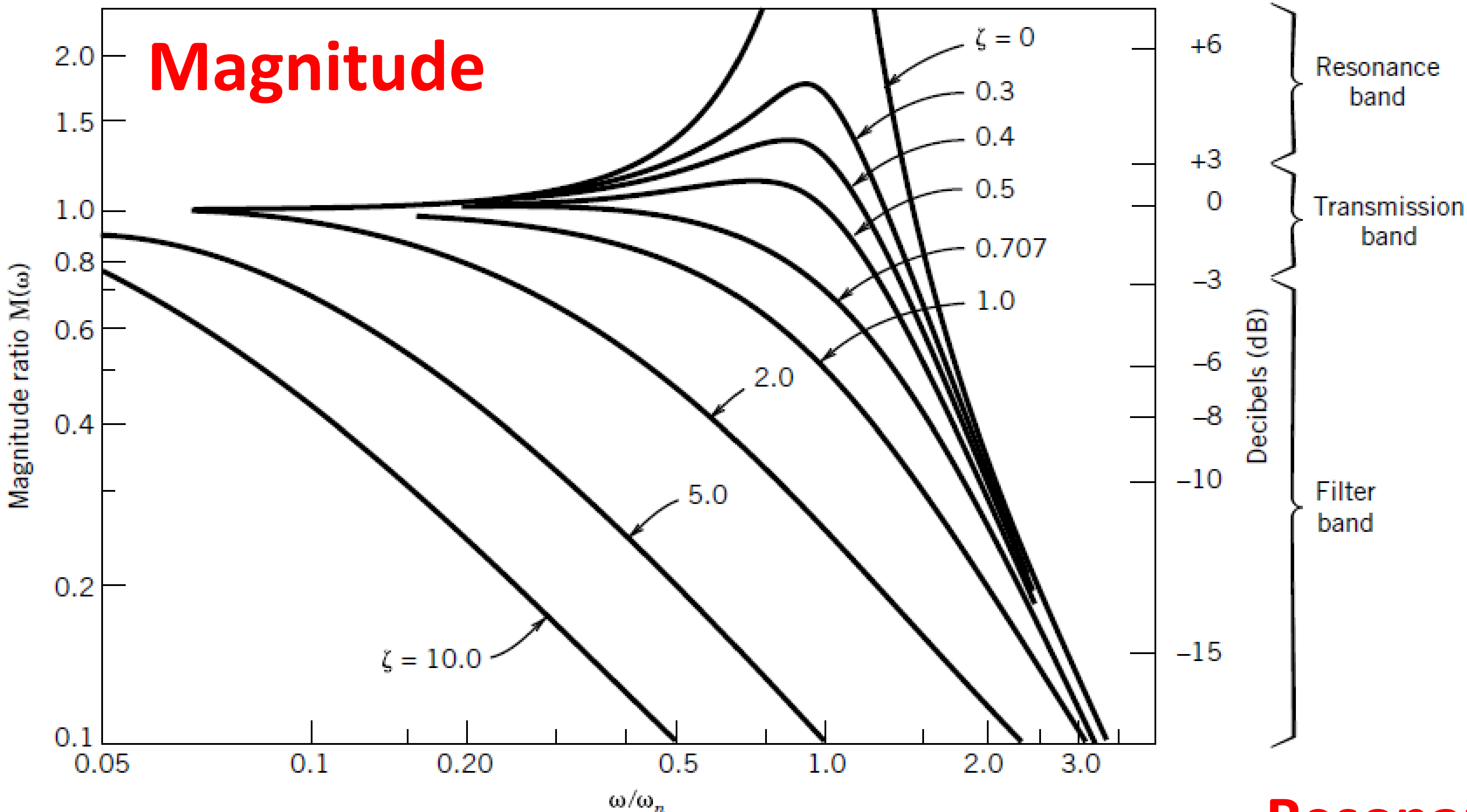


Figure 3.16 Second-order system frequency response: magnitude ratio.

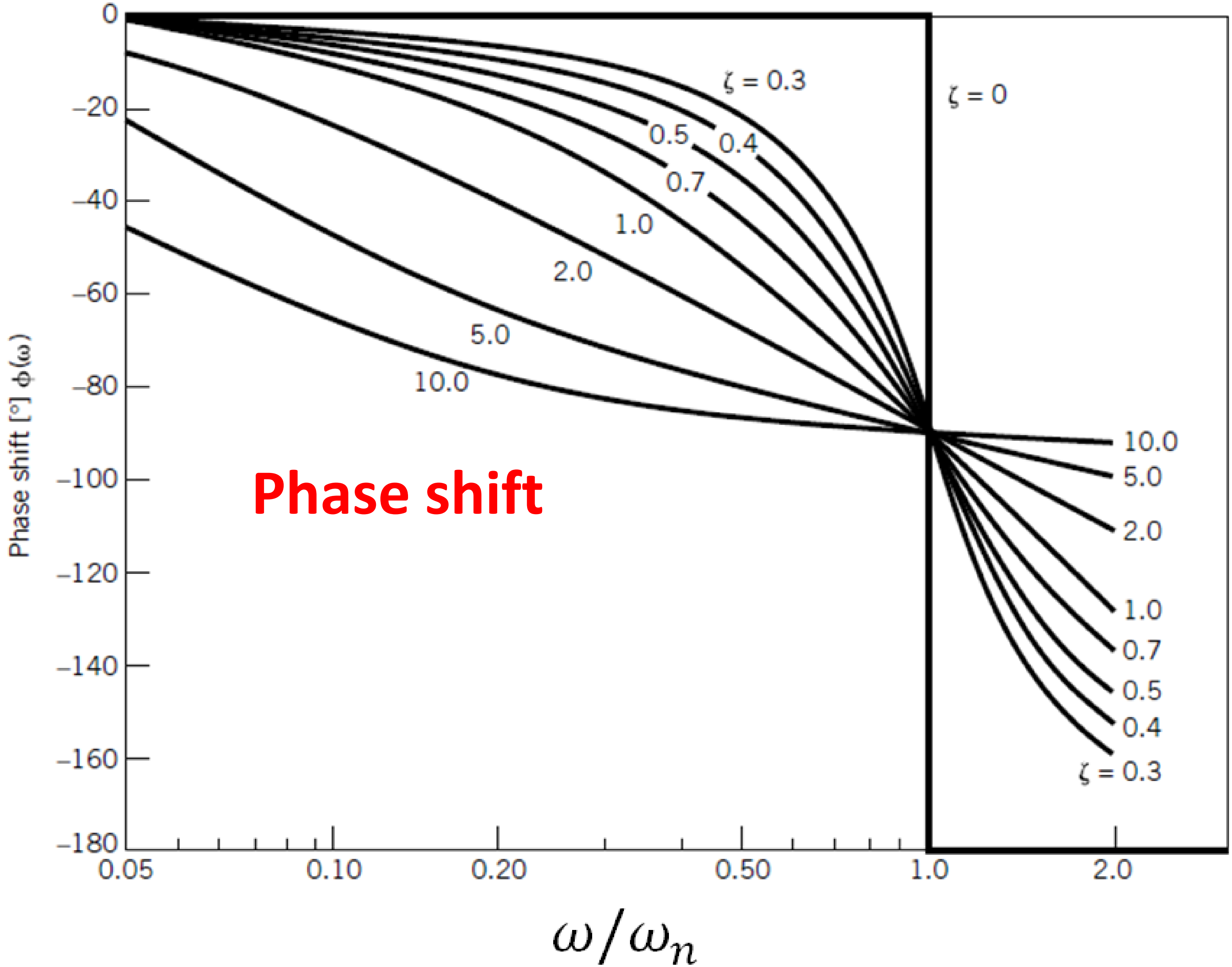
Resonance Frequency

$$\omega_R = \omega_n \sqrt{(1 - 2\zeta^2)}$$

$$\omega_R = \omega_n \sqrt{1 - 2\zeta^2}$$

Resonance frequency. Peak value

9b-frequency response



Illustrative examples

1-first order system with step input
2-first order system with periodic input
3-2nd order system with step input
4-2nd order system with periodic input

Illustrative examples

1-first order system with step input

A first order system for measuring temperature has a time constant of 5 s. The initial temperature is 25 ° C . The temperature is suddenly changed to 60 ° C.

a-Calculate the time when the temperature reaches 50° C.

b-What is the rise time to achieve 90% of the final value

We have the relation

Error Fraction function $\Gamma(t) = \dfrac{y(t) - y_\infty}{y_0 - y_\infty} = e^{-t/\tau}$

y(0) =y_o=25,

y(∞)=y_∞=60, y(t)=50 ,

τ=5 s

$$\frac{50-60}{25-60} = e^{-\frac{t}{5}} \qquad \frac{2}{7} = e^{-\frac{t}{5}}$$

$$\ln\left(\frac{2}{7}\right) = -\frac{t}{5} \qquad -1.25 = -\frac{t}{5}$$

or

t=6.26 s

Same thing for part b to get t=8.82 s

Illustrative examples

2-first order system with periodic input

Consider a first order system with time constant τ=1s. Find the magnitude ratio and phase shift for the following input frequencies

f (hz))	0.01	0.05	0.1	0.5	1	5	10	20

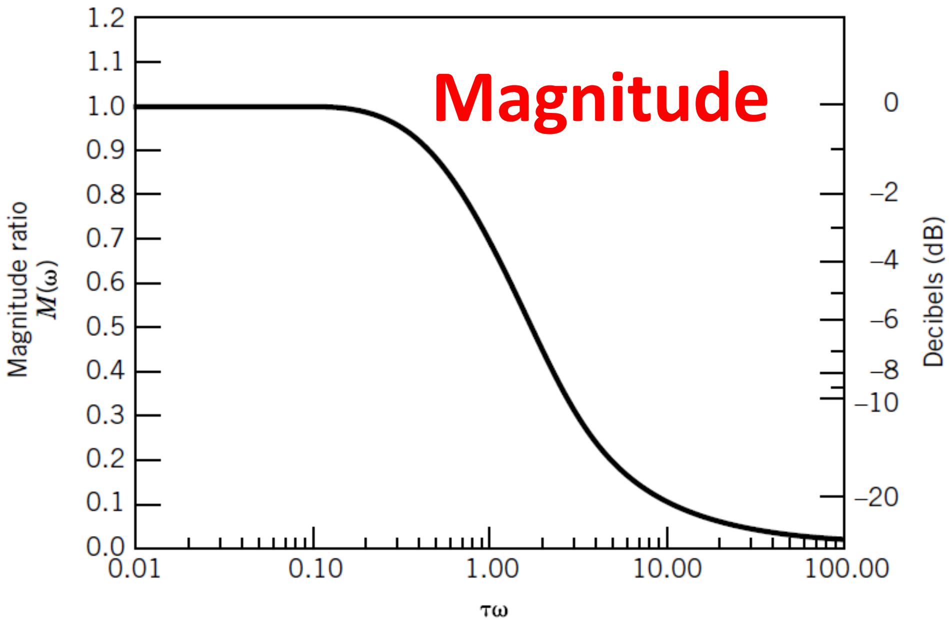
Magnitude $$M(\omega) = \frac{B}{KA} = \frac{1}{\sqrt{1 + (\omega\tau)^2}}$$

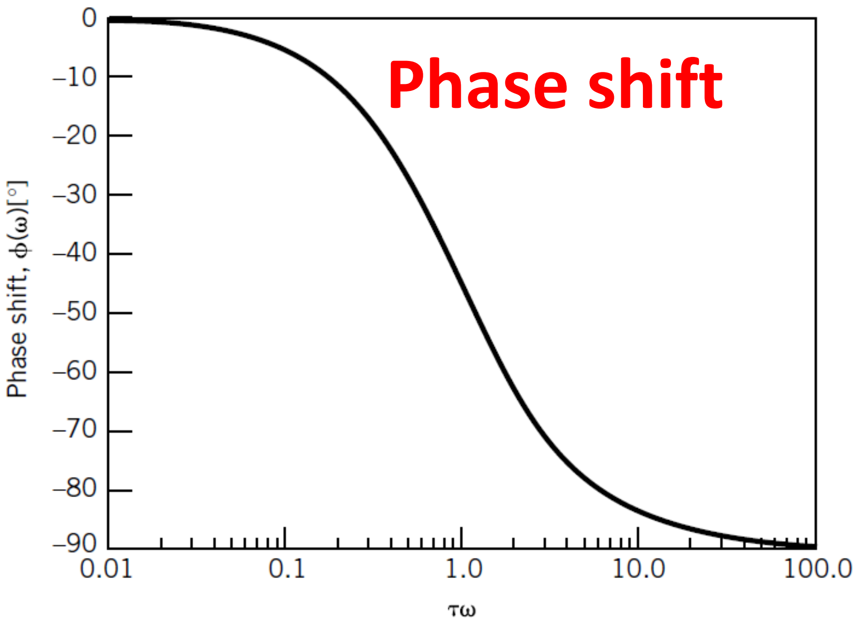
Phase shift $$\Phi = -\tan^{-1}(\omega\tau)$$

$$\omega = 2\pi f$$

f (Hz)	0.01	0.05	0.1	0.5	1.0	5	10
ω (rad/s)	0.06	0.31	0.63	6.28	12.57	31.42	62.83
$\omega\tau$	0.06	0.31	0.63	6.28	12.57	31.42	62.83
M(ω)	0.998	0.954	0.847	0.157	0.079	0.032	0.016
Φ (deg)	-3.6	-17.4	-32.1	-81.0	-85.5	-88.2	-89.1

8b-frequency response for first order system





Illustrative examples

3-2nd order system with step input

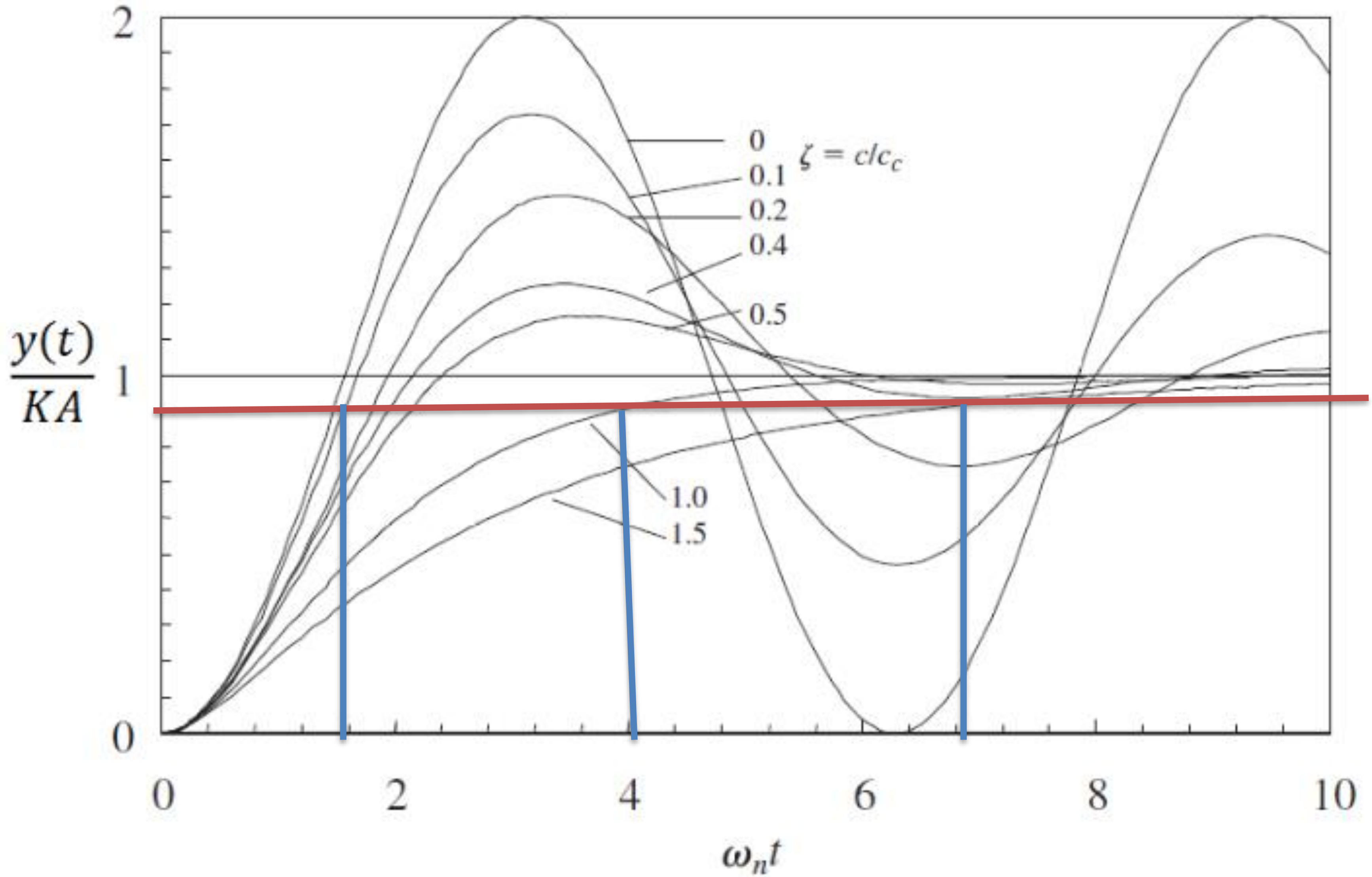
Consider a second order system with mass of 2 kg and a spring constant of k=100 N/m. How long it will take to achieve 90% of the unit step function for damping ratio of 0.1, 1.0, and 1.5

Natural frequency [Rad] $\omega_n = \sqrt{\frac{k}{m}} = 7.07\ rad/s$ Damping ratio= ξ

$$y(t) = KA - KAe^{-\zeta\omega_n t}\left[\frac{\zeta}{\sqrt{1-\zeta^2}}\sin\left(\omega_n t\sqrt{1-\zeta^2}\right) + \cos\left(\omega_n t\sqrt{1-\zeta^2}\right)\right] \quad 0 \le \zeta < 1 \quad (3.15a)$$

$$y(t) = KA - KA(1+\omega_n t)e^{-\omega_n t} \quad \zeta = 1 \quad (3.15b)$$

$$y(t) = KA - KA\left[\frac{\zeta + \sqrt{\zeta^2-1}}{2\sqrt{\zeta^2-1}}e^{\left(-\zeta+\sqrt{\zeta^2-1}\right)\omega_n t} + \frac{\zeta - \sqrt{\zeta^2-1}}{2\sqrt{\zeta^2-1}}e^{\left(-\zeta-\sqrt{\zeta^2-1}\right)\omega_n t}\right] \quad \zeta > 1 \quad (3.15c)$$



ξ	0.1	1.0	1.5		
$\omega_n t$	0.8	4	6.8		
T [s]	0.11	0.57	0.96		

Illustrative examples

4-2nd order system with periodic input

For a second order system with damping ratio of 0.3 and natural frequency of 10000 Hz. Determine the range of frequencies for which the **dynamic error** less than 10%.

$$\zeta = 0.3, \qquad \omega_n = 10000\ Hz = 62831\ rad/s$$

The Magnitude ratio is given by

$$M(\omega) = \frac{B(\omega)}{KA} = \frac{1}{\left\{\left[1 - (\omega/\omega_n)^2\right]^2 + \left[2\zeta\omega/\omega_n\right]^2\right\}^{1/2}}$$

Or one can use the figure

$\zeta = 0.3$

Dynamic error

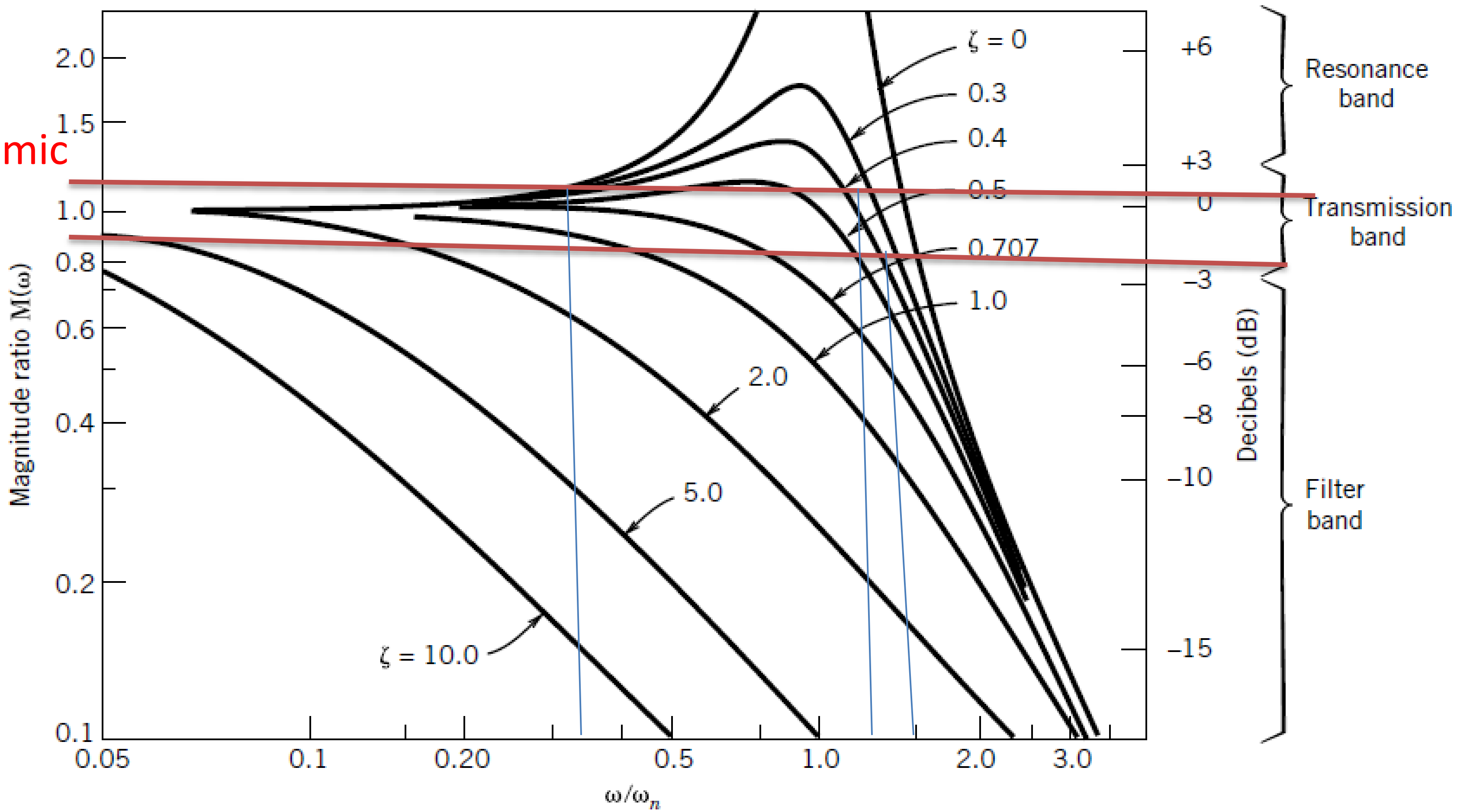


Figure 3.16 Second-order system frequency response: magnitude ratio.

From Magnitude ratio figure with dynamic ratio δ=10%

$$\frac{\omega}{\omega_n} = 0.35$$

$$\omega_n = 10000 * 2 * \pi = 62831$$

Then

$$\omega = 0.35 * \omega_n = 22000\,\frac{rad}{s} = 3501\ Hz$$

The magnitude ratio will be within 10% of the input value as long as $f < 3501$ Hz

الحمد للّه