

Simple iterative procedure to design un-finned double pipe heat exchanger

**1. Problem statement:**

A double pipe heat exchanger is to be sized (i.e. to find the area  $A_o$ , heat exchanger length  $L$ , and the inner pipe diameter  $d_i$ ) to meet a given heat load and not to exceed certain pressure drops in the tube and annulus sides. The mass flow rates and the temperatures for the two streams are given.

**2. Given data**

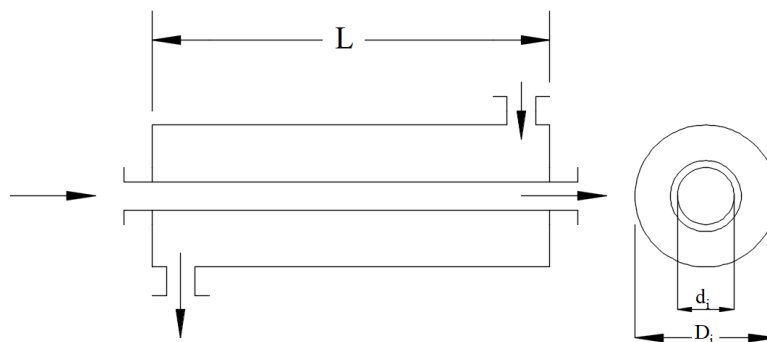
Tube side mass flow rate	Annulus side mass flow rate	Tube side fluid	Annulus side fluid	Inside fouling factor	Outside fouling factor	Tube wall thickness
$\dot{m}_t$	$\dot{m}_a$	√	√	$R_{fi}$	$R_{fo}$	$t$
[kg/s]	[kg/s]			[m <sup>2</sup> .K/W]	[m <sup>2</sup> .K/W]	[mm]

Inlet cold fluid temp.	Outlet cold fluid temp.	Inlet hot fluid temp.	Outlet hot fluid temp.	Tube side Max. allowable pressure drop	Annulus Max. allowable pressure drop
$T_{ci}$	$T_{co}$	$T_{hi}$	$T_{ho}$	$\Delta P_{t,max}$	$\Delta P_{a,max}$
[°C]	[°C]	[°C]	[°C]	[Pa]	[Pa]

Tube side operating pressure	Annulus side operating pressure	Which fluid in the tube side	Tube thermal conductivity	
$P_t$	$P_a$	√	$k_t$	
[Pa]	[Pa]		W/m.K	

**3. Assumptions**

Tube wall thickness	Max. tube velocity	Max. annulus velocity
$t$	$V_{t,max}$	$V_{a,max}$
[mm]	[m/s]	[m/s]



Double pipe heat exchanger

#### 4. Procedure

1- Calculate the fourth temperature if not given using heat load to be the same for the cold and hot side.

$$q = C_c \Delta T_c = C_h \Delta T_h \quad (1)$$

Then calculate the  $LMTD_{CF}$

$$LMTD_{CF} = \frac{(T_{ho}-T_{ci})-(T_{hi}-T_{co})}{\ln(T_{ho}-T_{ci})/(T_{hi}-T_{co})} \quad (2)$$

Also calculate the LMTD correction factor F if required.

2- Calculate the fluid properties at the mean temperatures

Property	Cold side	Hot side
Average Temp.	$T_{ca}$	$T_{ah}$
Density	$\rho_c$	$\rho_h$
Specific heat	$C_{pc}$	$C_{ph}$
Thermal conductivity	$k_c$	$k_h$
Viscosity	$\mu_c$	$\mu_h$
Prandtl number	$Pr_c$	$Pr_h$

3- Based on an assumed max. velocity  $V_{t,max}$  for tube side, get  $d_i$ .

$$\dot{m}_t = \rho_t V_{t,max} A_{ct} \quad (3)$$

where  $A_{ct}$  is the cross-section area of the tube.

$$A_{ct} = \frac{\pi}{4} d_i^2 \quad (4)$$

$V_{t,max}$  is the tube assumed maximum velocity. Assume typical wall thickness t get  $d_o$ .

4- Using the given mass flow rate in the annulus and the assumed annulus max velocity, calculate the inside diameter of the annulus  $D_i$ .

$$\dot{m}_a = \rho_a V_{a,max} A_{ca} \quad (5)$$

where the cross-section flow area of the annulus flow side is

$$A_{ca} = \frac{\pi}{4} (D_i^2 - d_o^2) \quad (6)$$

Calculate the equivalent diameter  $D_e$  for heat transfer calculations

$$D_e = \frac{4(\pi D_i^2/4 - \pi d_o^2/4)}{\pi d_o} = \frac{D_i^2 - d_o^2}{d_o} \quad (7)$$

Also calculate the hydraulic diameter  $D_h$  as follows

$$D_h = \frac{4A_{ca}}{P} = \frac{4A_{ca}}{\pi D_i + \pi d_o} = \frac{4\left(\frac{\pi}{4}(D_i^2 - d_o^2)\right)}{\pi(D_i + d_o)} = D_i - d_o \quad (8)$$

5- For tube side calculate  $Re_t$ ,  $Nu_t$  and  $h_t$

6-For annulus flow calculate  $Re_a$ ,  $Nu_a$ , and  $h_a$

7-You may assume typical value for the fouling resistances  $R_{fi}$ , and  $R_{fo}$

8-Calculate  $U_c$  and  $U_f$

$$\frac{1}{U_f} = \frac{1}{h_t(A_i/A_o)} + \frac{R_{fi}}{A_i/A_o} + \frac{A_o \ln(d_o/d_i)}{2\pi kL} + \frac{1}{h_a} + R_{fo} \quad (9)$$

9-Use the heat rate equation

$$q = U_f A_o LMTD.F \quad (10)$$

to get the heat exchanger length L, where  $A_o$  is given by

$$A_o = \pi d_o L \quad (11)$$

10-Calculate the pressure drops in tube size and the annulus flow

i.e.  $\Delta p_t$  and  $\Delta p_a$  using

$$\Delta P_t = 4f_t \frac{L}{d_i} \rho_t \frac{V_t^2}{2} \quad (12)$$

$$\Delta P_a = 4f_a \frac{L}{D_h} \rho_a \frac{V_a^2}{2} \quad (13)$$

where  $f_t$  and  $f_a$  are the friction coefficient (Kakac symbol for friction coefficient) for tube and annulus side respectively. For turbulent flow inside smooth pipes, it can be found using

$$f = [1.58 \ln(Re) - 3.28]^{-2} \quad (14)$$

For laminar flow

$$f = \frac{16}{Re} \quad (15)$$

11-Calculate the difference between the allowable maximum pressure and the pressure calculated from the previous step. It is called Residual Sum of Squares RSS

$$R_{ss} = \sqrt{(\Delta P_t - \Delta P_{t,max})^2 + (\Delta P_a - \Delta P_{a,max})^2} \quad (16)$$

When  $R_{ss}$  is higher than a prescribed value, one can start the iteration process by computing the tube and annulus velocity from the pressure drop for each side

11- Based on the required maximum allowable pressure drop for both the tube and annulus sides, calculate new values for the velocity in tube and in annulus as follows

$$\Delta P_{t,max} = 4f_t \frac{L}{d_i} \rho_t \frac{V_t^2}{2} \quad (15)$$

$$\Delta P_{a,max} = 4f_a \frac{L}{D_h} \rho_a \frac{V_a^2}{2} \quad (16)$$

use old values of  $d_i$ ,  $D_h$ ,  $f_t$  and  $f_a$ .

12- Continue the iterations until the convergence criterion for  $R_{ss}$  is met

$$V_t = [(\Delta P_{t,max}/4f_t)(d_i/L)(2/\rho_t)]^{0.5}$$

$$V_a = [(\Delta P_{a,max}/4f_a)(D_h/L)(2/\rho_a)]^{0.5}$$

#### 4. List of symbols

Symbols	meaning	
$A_{ca}$	Annulus cross section flow area	
$A_{ct}$	Tube cross section flow area	
$A_o$	Outside heat transfer area	
$d_i$	Outer pipe inside diameter	
$D_h$	Hydraulic diameter for Re and pressure drop calculations	
$D_e$	Equivalent annulus diameter for heat transfer calculations	
$d_i$	Inner pipe inside diameter	
$f_a$	Annulus side friction coefficient (Kakac symbol)	
$f_t$	Tube side friction coefficient (Kakac symbol)	
$h_a$	Annulus side heat transfer coefficient	
$h_t$	Tube side heat transfer coefficient	
$k_t$	Tube thermal conductivity	
$L$	Heat exchanger length	
$\dot{m}_a$	Annulus side flow rate	
$\dot{m}_t$	Tube side flow rate	
$R_{fi}$	Internal flow fouling resistance	
$R_{fo}$	External flow fouling resistance	
$t$	Inner pipe wall thickness	
$V_t$	Tube side flow velocity	
$V_a$	Annulus side flow velocity	
$V_{t,max}$	Max. Tube side flow velocity	
$V_{a,max}$	Max. Annulus side flow velocity	
$\Delta p_t$	Tube side pressure drop	
$\Delta p_a$	Annulus side pressure drop	
$\Delta p_{t,max}$	Max. Tube side pressure drop	
$\Delta p_{a,max}$	Max. Annulus side pressure drop	

Example

2. Given data

Tube side mass flow rate	Annulus side mass flow rate	Tube side fluid	Annulus side fluid	Inside fouling factor	Outside fouling factor	Tube wall thickness
$\dot{m}_t=1.36$	$\dot{m}_a=1.39$	water	water	$R_{fi}=0.00176$	$R_{fo}=0.00176$	$k_t=2$ mm
[kg/s]	[kg/s]			[m <sup>2</sup> .K/W]	[m <sup>2</sup> .K/W]	

Inlet cold fluid temp.	Outlet cold fluid temp.	Inlet hot fluid temp.	Outlet hot fluid temp.	Tube side Max. allowable pressure drop	Annulus Max. allowable pressure drop
$T_{ci}=20$	$T_{co}=35$	$T_{hi}=140$	$T_{ho}=125$	$\Delta P_{t,max}=850$	$\Delta P_{a,max}=4825$
[°C]	[°C]	[°C]	[°C]	[Pa]	[Pa]

Tube side operating pressure	Annulus side operating pressure	Which fluid in the tube side	Tube wall thermal conductivity	
$P_t=15$	$P_a=150$	hot fluid	$k_t=20$	
[MPa]	[kPa]		W/(m.K)	

Property	Cold side	Hot side
Average temperature	$T_{ca}=27.5$ °C	$T_{ha}=132.5$ °C
Density	$\rho_c=996.3$ kg/m <sup>3</sup>	$\rho_h=932.8$ kg/m <sup>3</sup>
Specific heat	$C_{pc}=4.183$ kJ/kg. K	$C_{ph}=4.272$ kJ/kg.K
Thermal conductivity	$k_c=0.5989$ W/m.K	$k_h=0.6697$ W/m.K
Viscosity	$\mu_c=8.42E-4$ Pa.s	$\mu_h=2.087E-4$ Pa.s
Prandtl number	$Pr_c=5.882$	$Pr_h=1.331$

Start the integrations by assuming maximum velocity in the tube and annulus side as 2.5 m/s. i.e.

$$V_{t,max}=2.5 \text{ m/s}$$

$$V_{a,max}=2.5 \text{ m/s}$$

	$V_t$	$V_a$	$d_i$	$Re_{di}$	$Re_{Dh}$	$h_t$	$h_a$	$U_f$	$A_o$	$L$	$\Delta P_t$	$\Delta P_a$	$R_{ss}$
	[m/s]	[m/s]	[m]	[-]	[-]	W/m <sup>2</sup> .K	W/m <sup>2</sup> .K	W/m <sup>2</sup> .K	[m <sup>2</sup> ]	[m]	[Pa]	[Pa]	
1	2.5	2.5	0.02725	304524	29060	15011	4579	1283	0.6477	6.6	10158	49813	45941
2	0.7232	0.7781	0.0507	163788	16514	4916	1586	758	1.096	6.4	499.4	2950	1908
3	0.9435	0.9952	0.04436	187078	18670	6245	1980	868	0.957	6.3	932.8	5218	401
4	0.9006	0.957	0.0454	182775	18284	5989	1913	850	0.977	6.3	833.4	4761	66
5	0.909	0.963											