Important correlations figures and tables for MEP365 Thermal Measurements

2021

Ch.1 Introduction

1.1 Instrument uncertainty

$$u_c = \sqrt{\left(e_1{}^2 + e_2{}^2 + e_3{}^2 + \ldots. e_m{}^2\right)}$$

Where e_1, e_2, ….. are the errors

Ch. 2 & Ch. 3 Signals & Response of a Measurement System

A) Signals

<table>
<tr><th>Deterministic functions</th><th>Non-deterministic functions</th></tr>
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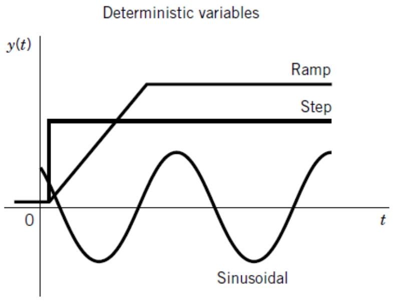

Figure 2.5 Examples of dynamic signals.
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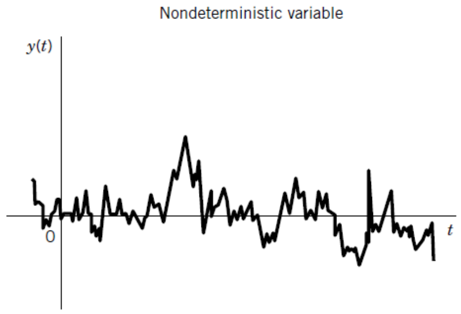

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Signal average and RMS (Root Mean Squared)

Average $$\bar{y} = \frac{\int_{t_1}^{t_2} y(t)dt}{\int_{t_1}^{t_2} dt}$$

RMS (Root Mean Squared) $$y_{rms} = \sqrt{\frac{1}{t_2 - t_1}\int_{t_1}^{t_2} y^2 dt}$$

Sinusoidal wave

$$y(t) = Acos(\omega t) + Bsin(\omega t)$$

Period T [s] & frequency *f* [Hz]

$$T = \frac{2\pi}{\omega} = \frac{1}{f}$$

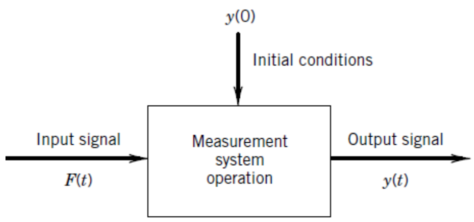
The combined sine and cosine function can be written in either sine or cosine wave:

$$y(t) = A\cos(\omega t) + B\sin(\omega t)$$
$$y(t) = C\cos(\omega t - \phi)$$
$$y(t) = C\sin(\omega t + \phi^*)$$

where

$C = \sqrt{A^2 + B^2}$ and $\phi = \tan^{-1}\left(\frac{B}{A}\right)$, $\phi^* = \tan^{-1}\left(\frac{A}{B}\right)$, $\phi^* = \frac{\pi}{2} - \phi$

B) System response



General form of measuring system differential equation

$$a_n \frac{d^n y}{dt^n} + a_{n-1}\frac{d^{n-1}y}{dt^{n-1}} + \cdots a_1 \frac{dy}{dt} + a_0\, y = F(t)$$

B1- zero order system

$$a_0 y = F(t)$$
$$y(t) = KF(t)$$

K=1/a_0 is called static sensitivity

B2-First order system

$$a_1 \frac{dy}{dt} + a_0 y = F(t)$$
$$\frac{a_1}{a_0}\frac{dy}{dt} + y = \frac{1}{a_0}\, F(t)$$
$$\tau \frac{dy}{dt} + y = KF(t)$$

τ is the time constant, which a fundamental characteristic of a first order system

B2-a step response for first order system

Input step: $F(t) = AU(t)$

$$\tau \dot{y} + y = KF(t) = KAU(t)$$

U(t) is the unit step

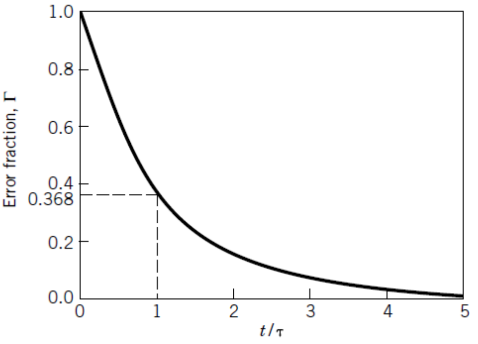
The solution is given by:

$$y(t) = KA \quad + \quad (y_0 - KA)e^{-t/\tau}$$

Steady **Transient part**

The error fraction function is defined as

$$\Gamma(t) = \frac{y(t) - y_\infty}{y_0 - y_\infty} = e^{-t/\tau}$$



B2-b Frequency response for the first order system

$$\tau \dot{y} + y = KAsin(\omega t)$$

Transfer function

$$G(s) = \frac{1}{1 + \tau s}$$

The general solution is given by:

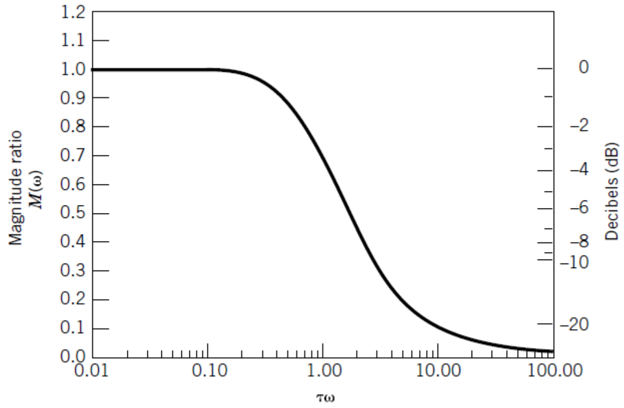
$$y(t) = Ce^{-t/\tau} + \frac{KA}{\sqrt{1 + (\omega t)^2}} \sin\left(\omega t - \tan^{-1} \omega t\right)$$

$$y(t) = Ce^{-t/\tau} + B(\omega)\sin[\omega t + \Phi]$$
$$B(\omega) = \frac{KA}{\sqrt{1 + (\omega\tau)^2}}$$
$$\Phi(\omega) = -\tan^{-1}(\omega\tau)$$

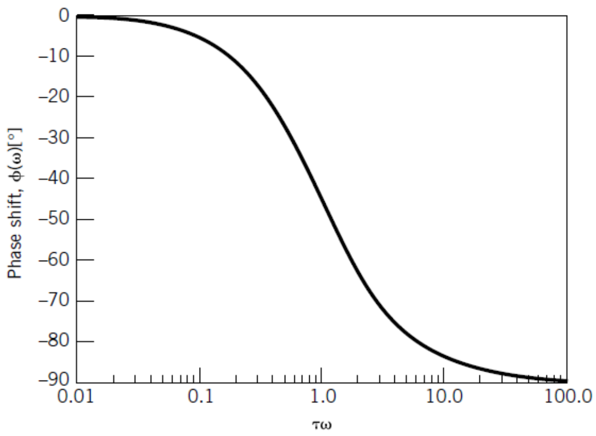
Magnitude $M(\omega) = \frac{B}{KA} = \frac{1}{\sqrt{1+(\omega\tau)^2}}$

Time delay β₁, $\beta_1 = \frac{\phi}{\omega}$

Magnitude



Phase shift



B3-Second order system

$$m\ddot{y} + c\dot{y} + ky = F(t)$$

$$a_2\ddot{y} + a_1\dot{y} + a_0 y = F(t)$$

$$\frac{1}{\omega_n^2}\ddot{y} + \frac{2\zeta}{\omega_n}\dot{y} + y = KF(t)$$

Natural frequency $$\omega_n = \sqrt{\frac{a_0}{a_2}} = \sqrt{\frac{k}{m}}$$

Damping ratio $$\zeta = \frac{c}{c_c} = \frac{a_1}{2\sqrt{a_0 a_2}} = \frac{c}{2\sqrt{km}}$$

B3-a step response for a second order system

$$y(t) = KA - KAe^{-\zeta\omega_n t}\left[\frac{\zeta}{\sqrt{1-\zeta^2}}\sin\left(\omega_n t\sqrt{1-\zeta^2}\right) + \cos\left(\omega_n t\sqrt{1-\zeta^2}\right)\right] \quad 0 \le \zeta < 1$$

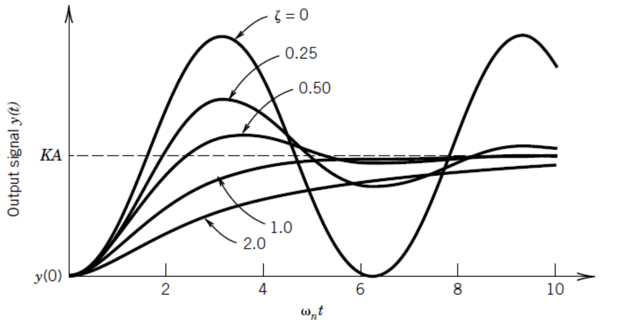
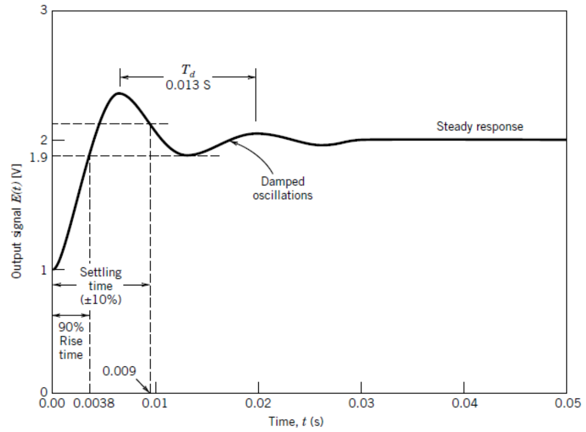
$$y(t) = KA - KA(1 + \omega_n t)e^{-\omega_n t} \quad \zeta = 1$$

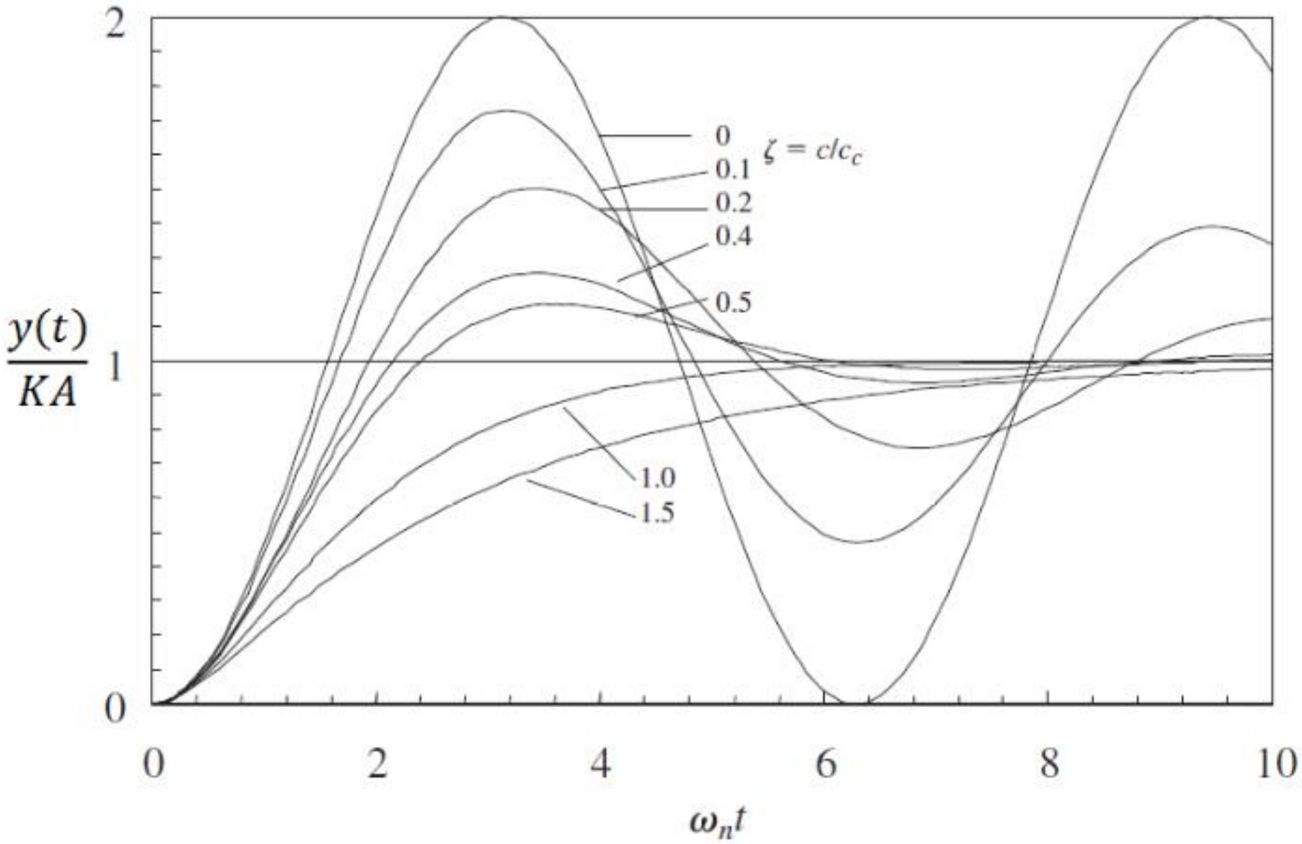
$$y(t) = KA - KA\left[\frac{\zeta + \sqrt{\zeta^2-1}}{2\sqrt{\zeta^2-1}}e^{\left(-\zeta+\sqrt{\zeta^2-1}\right)\omega_n t} + \frac{\zeta - \sqrt{\zeta^2-1}}{2\sqrt{\zeta^2-1}}e^{\left(-\zeta-\sqrt{\zeta^2-1}\right)\omega_n t}\right] \quad \zeta > 1$$

Ringing frequency

$$T_d = \frac{2\pi}{\omega_d} = \frac{1}{f_d}$$

$$\omega_d = \omega_n\sqrt{1-\zeta^2}$$

Step response	Rise time, settling time and Ringing frequency for under damped system
 **Figure 3.14** Second-order system time response to a step function input.	



B3-b Frequency Response for the second order system due periodic input

Transfer function

$$G(j\omega) = \frac{1}{\left[1 - \left(\frac{\omega}{\omega_n}\right)^2 + \left(\frac{2\zeta\omega}{\omega_n}\right) j\right]}$$

Magnitude: $$M(\omega) = \frac{B(\omega)}{KA} = \frac{1}{\left\{\left[1 - \left(\frac{\omega}{\omega_n}\right)^2\right]^2 + [2\zeta\omega/\omega_n]^2\right\}^{1/2}}$$

Phase shift: $$\phi(\omega) = \tan^{-1}\left(-\frac{2\zeta\omega/\omega_n}{1 - \left(\frac{\omega}{\omega_n}\right)^2}\right)$$

Resonance frequency $\omega_R = \omega_n\sqrt{1 - 2\zeta^2}$

Dynamic error $$\delta(\omega) = M(\omega) - 1$$

Magnitude

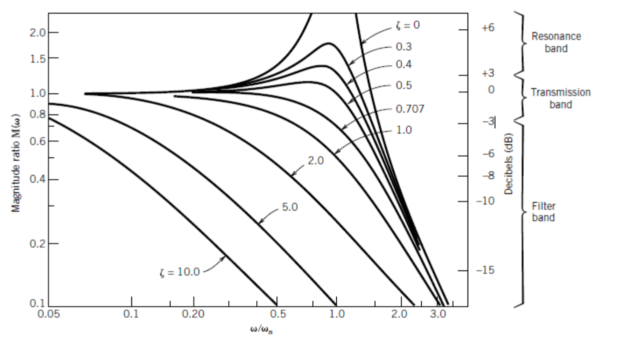


Figure 3.16 Second-order system frequency response: magnitude ratio.

Phase shift

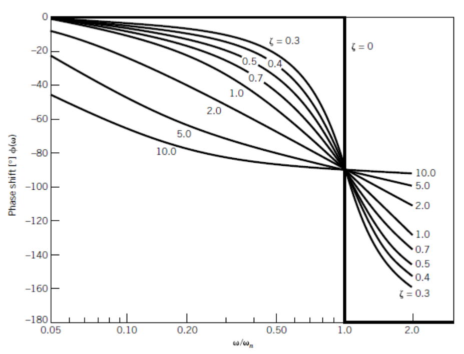


Figure 3.17 Second-order system frequency response: phase shift.

Ch. 4 Probability and Statistics

if x' is the true value, $\bar{x}$ is the mean value and $u_{\bar{x}}$ is the uncertainty then the true value for certain probability is given by

$$x' = \bar{x} \pm u_{\bar{x}} \qquad (P\%)$$

Number of intervals K to generate frequency distribution

$K = 1.87(N-1)^{0.4} + 1$ N is the number of data points. For very large value of N, use $K = N^{\frac{1}{2}}$ provided at least one interval with occurrences ≥ 5 (i.e. $n_j \geq 5$).

4.1 Infinite statistics

If the probability density function p(x) is known in the absence of the systematic errors ($x' = \bar{x}$), then the true mean value can be found using

$$x' = \int_{-\infty}^{+\infty} x p(x) dx$$

The variance is given by

$$\sigma^2 = \int_{-\infty}^{+\infty} (x - \bar{x})^2 p(x) dx$$

and the standard of deviation is σ

Normal (Gauss normal distribution function)

$$p(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2} \frac{(x - x')}{\sigma^2} \right]$$

Define z_1 as

$$z_1 = \frac{x_1 - x'}{\sigma} \text{ and } \beta = \frac{\text{x - x'}}{\sigma}$$

Probability for z to be between $-z_1$ and z_1

$$P(-z_1 \leq z \leq +z_1) = 2\left[\frac{1}{\sqrt{2\pi}} \int_0^{z_1} e^{-\beta^2/2} d\beta \right]$$

The term between the two brackets above is called half sided integral. It is tabulated in the following table (Table 4.3)

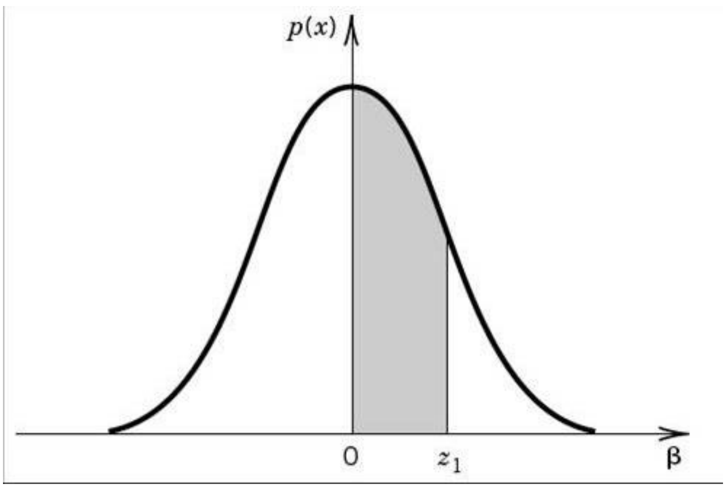
The probability that the i^{th} measured value will have a value in the range $x' \pm z_1\sigma$ is $2P(z_1)*100$ =P%

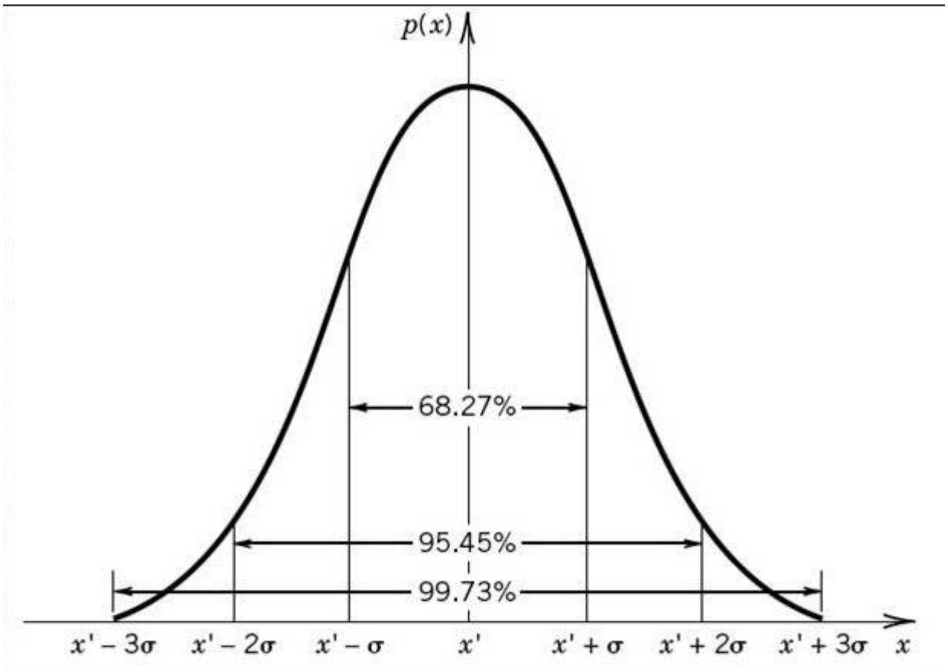
$$x_i = x' \pm z_1\sigma \qquad \text{(P\%)}$$

Table 4.3 Probability Values for Normal Error Function

One-Sided Integral Solutions for $p(z_1) = \frac{1}{(2\pi)^{1/2}} \int_0^{z_1} e^{-\beta^2/2} d\beta$

$z_1 = \frac{x_1 - x'}{\sigma}$	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1809	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4758	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4799	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.49865	0.4987	0.4987	0.4988	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990





4.2 Finite statistics

Sample mean, $\bar{x}$

$$\bar{x} = \frac{1}{N}\sum_{i=1}^{N} x_i$$

Sample variance, $s_x{}^2$

$$s_x{}^2 = \frac{1}{N-1}\sum_{i=1}^{N}(x_i - \bar{x})^2$$

Sample standard of deviation, s_x

$$s_x = \sqrt{s_x{}^2}$$

Standard deviation of the mean

$$s_{\bar{x}} = \frac{s_x}{\sqrt{N}}$$

Sample data interval for certain probability $\bar{x} \pm t_{v,P} s_x \quad (P\%)$

True mean value estimation with probability P(%) $\bar{x} \pm t_{v,P} s_{\bar{x}} \quad (P\%)$

$t_{v,P}$ is the t-estimator which can be found from table 4.4 below.
v is the degree of freedom =N-1

Table 4.4 Student's *t* Distribution

ν	t_{50}	t_{90}	t_{95}	t_{99}
1	1.000	6.314	12.706	63.657
2	0.816	2.920	4.303	9.925
3	0.765	2.353	3.182	5.841
4	0.741	2.132	2.770	4.604
5	0.727	2.015	2.571	4.032
6	0.718	1.943	2.447	3.707
7	0.711	1.895	2.365	3.499
8	0.706	1.860	2.306	3.355
9	0.703	1.833	2.262	3.250
10	0.700	1.812	2.228	3.169
11	0.697	1.796	2.201	3.106
12	0.695	1.782	2.179	3.055
13	0.694	1.771	2.160	3.012
14	0.692	1.761	2.145	2.977
15	0.691	1.753	2.131	2.947
16	0.690	1.746	2.120	2.921
17	0.689	1.740	2.110	2.898
18	0.688	1.734	2.101	2.878
19	0.688	1.729	2.093	2.861
20	0.687	1.725	2.086	2.845
21	0.686	1.721	2.080	2.831
30	0.683	1.697	2.042	2.750
40	0.681	1.684	2.021	2.704
50	0.680	1.679	2.010	2.679
60	0.679	1.671	2.000	2.660
∞	0.674	1.645	1.960	2.576

Chauvenet's criterion for outlier data

Let z_0 be $z_0 = \left|\frac{x_i - \bar{x}}{s_x}\right|$

If $\left(1 - 2 * P(z_0)\right) < \frac{1}{2N}$ then it can be considered outlier

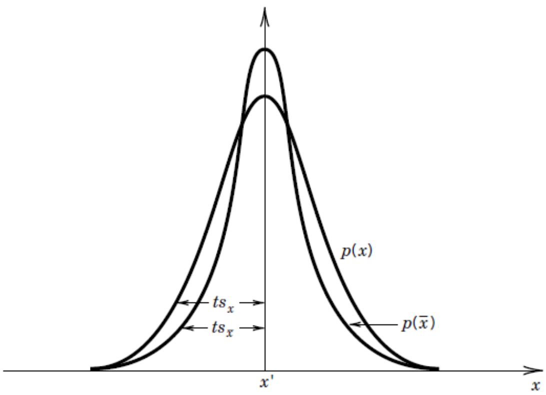
Number of measurements required

$$N_T \approx \left(\frac{t_{N_1-1,95} S_1}{d}\right)^2 \quad P=95\%$$

Additional data needed N_T-N_1

d=CI/2 where

CI is the confidence interval



Least squares method

A polynomial of order m between y and x is given by:

$$y_c = a_0 + a_1 x + a_2 x^2 + \ldots\ldots + a_m x^m$$

$$D = \sum_{i=1}^{N} \left[y_i - \left(a_0 + a_1 x + a_2 x^2 + \ldots a_m x^m \right) \right]^2$$

$$\frac{\partial D}{\partial a_0} = 0 = \frac{\partial}{\partial a_0} \left\{ \sum_{i=1}^{N} \left[y_i - (a_0 + a_1 x + a_2 x^2 + \ldots a_m x^m) \right]^2 \right\}$$

$$\frac{\partial D}{\partial a_1} = 0 = \frac{\partial}{\partial a_1} \left\{ \sum_{i=1}^{N} \left[y_i - (a_0 + a_1 x + a_2 x^2 + \ldots a_m x^m) \right]^2 \right\}$$

$$\frac{\partial D}{\partial a_2} = 0 = \frac{\partial}{\partial a_2} \left\{ \sum_{i=1}^{N} \left[y_i - (a_0 + a_1 x + a_2 x^2 + \ldots a_m x^m) \right]^2 \right\}$$

……..

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$$\begin{bmatrix} N & \sum_{i=1}^{N} x_i & \sum_{i=1}^{N} x_i^2 \\ \sum_{i=1}^{N} x_i & \sum_{i=1}^{N} x_i^2 & \sum_{i=1}^{N} x_i^3 \\ \sum_{i=1}^{N} x_i^2 & \sum_{i=1}^{N} x_i^3 & \sum_{i=1}^{N} x_i^4 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^{N} y_i \\ \sum_{i=1}^{N} x_i y_i \\ \sum_{i=1}^{N} x_i^2 y_i \end{bmatrix}$$

Correlation coefficient,

$$r = \sqrt{1 - \frac{s_{yx}^2}{s_y^2}}$$

Coefficient of determination r^2

Standard error of the fit: $s_{yx} = \sqrt{\dfrac{\sum_i^N (y_i - y_{ci})^2}{v}}$

Degree of freedom: $v = N - (m+1)$

Uncertainty of the fit $u = \pm t_{v,P} \dfrac{s_{yx}}{\sqrt{N}}$

Ch. 5 Uncertainty

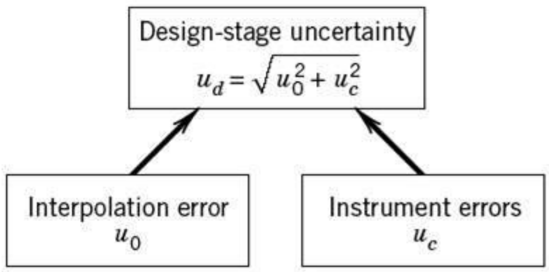
$$x' = \bar{x} \pm u_x \qquad (P\%)$$

Design stage uncertainty

$$u_d = \sqrt{\left[u_o^2 + u_c^2 \right]}$$

u_o=interpolation error=(1/2) resolution

u_c=instrumental error



Error Propagation

R=R(x_1,x_2,x_3,…..x_L)

$$u_R = \pm \left[\sum_{i=1}^{L} (\theta_i u_{xi})^2 \right]^{1/2} \qquad \theta_i = \left(\frac{\partial R}{\partial x_i} \right)_{x_i = \bar{x}_i}$$

Error Propagation using Numerical Approach

$$R = R(x_1, x_2, x_3, \ldots\ldots x_L) \qquad R_o = R(\bar{x}_1, \bar{x}_2, \bar{x}_3, \ldots\ldots \bar{x}_L)$$

$$R_i^+ = R(x_i + u_{x1}, x_2, x_3, \ldots\ldots x_L) \qquad R_i^- = R(x_i - u_{x1}, x_2, x_3, \ldots\ldots x_L)$$

$$\delta R_i^+ = R_i^+ - R_o \qquad \delta R_i^- = R_i^- - R_o$$

$$\delta R_i = \frac{\delta R_i^+ - \delta R_i^-}{2} = \theta_i u_i \qquad u_R = \pm \left[\sum_{i=1}^{L} \left(\delta R_i \right)^2 \right]^{1/2}$$

Procedure to find the uncertainty for multiple measurements based on grouping the elemental errors into Bias uncertainty b, and random uncertainty s

1-Perfom multiple measurements for x
2-Identfy elemental errors e_k
3-For each e_k assign $(b_{\bar{x}})_k$ and $(s_{\bar{x}})_k$
4-For each measurement, the **standard random uncertainty** is given by

$$s_{\bar{x}} = \frac{s_x}{\sqrt{N}}$$

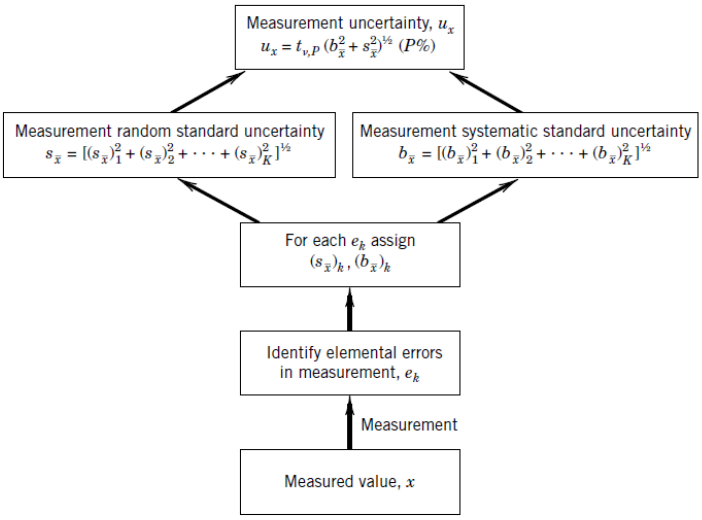


Figure 5.6 Multiple-measurement uncertainty procedure for combining uncertainties.

5-Combining the systematic and the random uncertainties into

$$b_{\bar{x}} = [(b_{\bar{x}}^2)_1 + (b_{\bar{x}}^2)_2 + (b_{\bar{x}}^2)_3 + \cdots (b_{\bar{x}}^2)_k]^{\frac{1}{2}}$$

$$s_{\bar{x}} = [(s_{\bar{x}}^2)_1 + (s_{\bar{x}}^2)_2 + (s_{\bar{x}}^2)_3 + \cdots . (s_{\bar{x}}^2)_k]^{\frac{1}{2}}$$

6-The expanded uncertainty is evaluated using

$$u_x = t_{v,P}[(b_{\bar{x}})^2 + (s_{\bar{x}})^2]^{\frac{1}{2}}$$

where the degree of freedom is found using

$$\nu = \frac{\left(\sum_{k=1}^{K} \left(s_{\bar{x}}^2\right)_k + \left(b_{\bar{x}}^2\right)_k \right)^2}{\sum_{k=1}^{K} \left(\left(s_{\bar{x}}^4\right)_k / \nu_k \right) + \sum_{k=1}^{K} \left(\left(b_{\bar{x}}^4\right)_k / \nu_k \right)}$$

The systematic part can be neglected in the above equation if it is very small.

Propagation of uncertainty to the results using the concept of grouping the errors into systematic and random errors

$$R' = \overline{R} \pm u_R \qquad \text{(P\%)}$$

$$u_R = f_2(b_{\bar{x}1}, b_{\bar{x}2}, +b_{\bar{x}3}, .. b_{\bar{x}L}; s_{\bar{x}1}, s_{\bar{x}2}, ... s_{\bar{x}L})$$

$$s_R = \left(\sum_{i=1}^{L} [\theta_i s_{\bar{x}i}]^2 \right)^{1/2} \qquad b_R = \left(\sum_{i=1}^{L} [\theta_i b_{\bar{x}i}]^2 \right)^{1/2} \qquad \theta_i = \left. \frac{\partial R}{\partial x_i} \right|_{x=\bar{x}}$$

$$\nu_R = \frac{\left\{ \sum_{i=1}^{L} (\theta_i s_{\bar{x}i})^2 \right\}^2}{\sum_{i-1}^{L} \left\{ (\theta_i s_{\bar{x}i})^4 / \nu_{\bar{x}i} \right\}}$$

$$u_R = t_{v,P}[b_R^2 + s_R^2]^{\frac{1}{2}}$$

Ch. 8 Temperature measurements

RTD

$R = R_o\left[1+\alpha(T-T_o)\right]$

For Platinum, α is 0.003927 C^{-1}

Thermistors

$R = R_o e^{\beta[1/T-1/T_o]}$

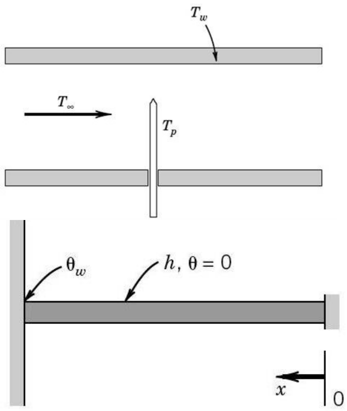
Typical values of β are between 3500 K and 4600 K.

Thermocouple

- Seebeck effect
- Peltier effect
- Thomson effect

Tables for the variation of emf from standard thermocouple with 0°C reference junction are given at the end of these sheets. Temperatures are in °C and emf in mV.

Conduction errors

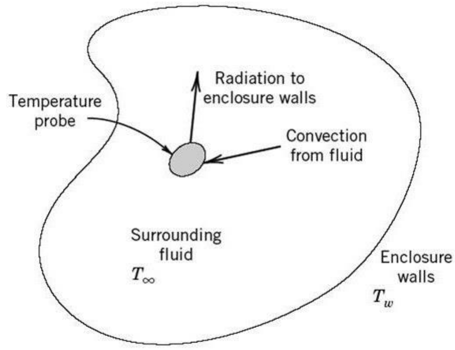


$\frac{\theta(x)}{\theta_w} = \frac{\cosh(mx)}{\cosh(mL)}$ where $m^2 = \frac{hP}{kA}$

$\frac{T_p - T_\infty}{T_w - T_\infty} = \frac{1}{Cosh(mL)}$

Conduction error, $e_c = T_p - T_\infty = \frac{T_w - T_\infty}{\cosh(mL)}$

Radiation errors



At equilibrium: heat by convection=heat by radiation

$q_c = q_r$

$hA_p(T_\infty - T_p) = FA_p\varepsilon_p\sigma\left[T_p^4 - T_w^4\right]$

σ=5.669*10^{-8} W/(m^2.K^4)

Radiation error $e_r = T_p - T_\infty$

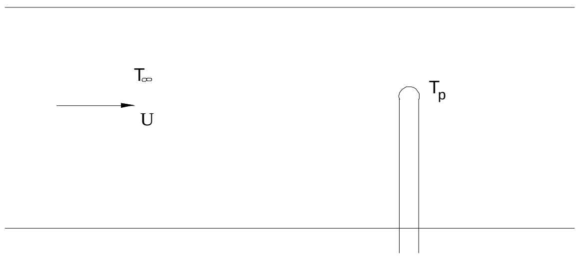
Newton-Raphson's method for solving non-linear equations

$T_{p,i+1} = T_{p,i} - \frac{f}{f'}$

Radiation error:

$e_r = T_p - T_\infty = \frac{F\varepsilon\sigma}{h}(T_w^4 - T_p^4)$

Recovery error (High speed flows)



Sound of speed in air is given by

$$a = \sqrt{kRT}$$

where
k is the specific heat ratio C_p/C_v
R ideal gas constant =287 J/kg.K
T is the temperature of air in Kelvin

Recovery error $\quad e_U = T_p - T_\infty = \dfrac{rU^2}{2C_p}$

for wires normal to flow $\quad r = 0.68 + 0.07$

for wires parallel to flow $\quad r = 0.86 + 0.09$

Relation between probe temperature T_p and stagnation temperature T_t

$$T_p = T_t - \frac{(1-r)U^2}{2C_p}$$

$$e_U = T_t - \frac{(1-r)U^2}{2C_p}$$

U is the flow speed
T_t is stagnation temperature (total temperature), which can be found using

$$\frac{U^2}{2} = C_p(T_t - T_\infty)$$

C_p must be in J/kg.K.

Transient behavior of a temperature sensor

Time constant τ is given by $\quad \tau = \dfrac{\rho \forall C_p}{hA}$
where
$\forall$ is the volume of the sensor (or probe)
ρ is the sensor (or probe) density
C_p sensor specific heat
h is the heat transfer between the sensor and the surrounding environment
A is the surface area of the sensor
For a probe initially at $T=T_i$, subjected to environment at T_∞

$$\tau \frac{dT}{dt} = (T_\infty - T)$$

or using $\theta=(T-T_i)$, and $\theta_\infty=(T_\infty-T_i)$

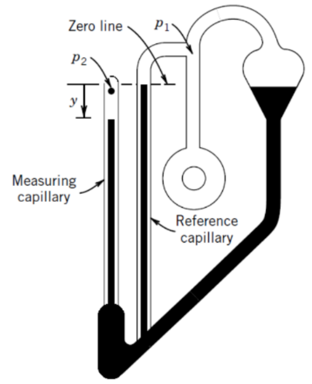
$$\frac{d\theta}{dt} + \frac{\theta}{\tau} = \frac{\theta}{\tau}$$

and the solution

$$\theta = \theta_\infty (1 - e^{-t/\tau})$$

Ch. 9 Pressure Measurements

γ=Specific weight=ρg [N/m^3]
S=Specific gravity=ρ/ρ_w [Dimensionless].



McLeod Gauge $P_1 = \frac{\gamma y^2 A}{V_1 - yA}$

Straight U tube manometer

$$\Delta p = (\rho_m - \rho) g H = (\gamma_m - \gamma) H$$

ρ_m is the manometer fluid density, and ρ is the fluid density

Inclined manometer

$$\Delta p = (\rho_m - \rho) g L \sin(\theta) = (\gamma_m - \gamma) L \sin(\theta)$$

θ is the inclined angle of the manometer with the horizontal

Deadweight tester

Gravity error for elevation z (in meter), and latitude angle ϕ (in degrees)

$$e_1 = -(2.637 * 10^{-3} \cos(2\phi) + 2.9 * 10^{-5} z + 5 * 10^{-5})$$

Buoyancy effect

$$e_2 = -\gamma_{air} / \gamma_{masses}$$

The indicated pressure is corrected using

$$p = p_i (1 + e_1 + e_2)$$

Pitot static tube

$$p_v = p_t - p_x = \frac{1}{2} \rho U_x^2 \quad \text{or} \quad U_x = \sqrt{\frac{2\Delta p}{\rho}}$$

For high speed gas

$$U = \sqrt{2[k/(k-1)][(p/\rho)^{(k-1)/k} - 1]}$$

k is specific heat ratio (C_p/C_v)

$$f_o = \frac{c \pm V_o}{c \mp V_s} f_s$$

Doppler effect
f_o observer frequency, f_s source frequency, V_o observer speed, V_s source speed, c=speed of sound or light

Thermal Anemometry

$$E^2 = C + DU^n$$

or

$$E_1 = KU$$

Doppler Anemometry

$$f_s = f_i + f_D$$

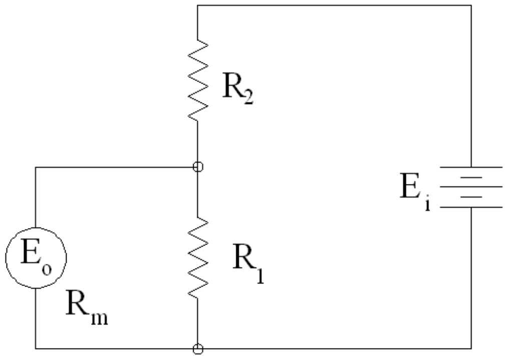
$$U = \frac{\lambda}{2 \sin(\theta/2)} f_D$$

Loading error

$$\frac{E_o}{E_i} = \frac{1}{1 + (R_2/R_1)[R_1/R_m + 1]}$$

When $R_m \to \infty$

$$\frac{E_o}{E_i} = \frac{R_1}{R_1 + R_2}$$



Ch. 10 Flow measurements

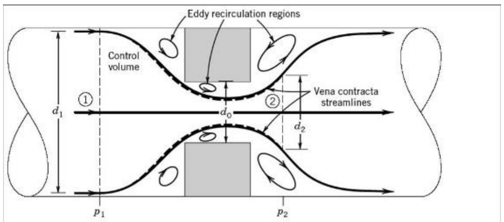
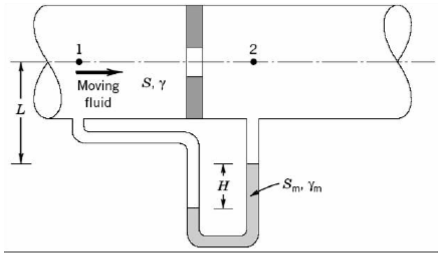


Figure 10.4 Control volume concept as applied between two streamlines for flow through an obstruction meter.



Flow rate through velocity determination

$$Q = \iint_A U dA$$

for circular pipe $Q = 2\pi \sum U_{ij} r \Delta r$

Obstruction meters (orifice, venturi, and nozzle)

$$Q_I = CEA_o \sqrt{\frac{2\Delta p}{\rho}} = K_o A_o \sqrt{\frac{2\Delta p}{\rho}}$$

where

Q_I is the volume flow rate assuming the flow to be incompressible

Velocity approach factor $E = \frac{1}{\sqrt{1-\beta^4}}$, $\beta = d_o / d_1$

C is the discharge coefficient=f(Re_{d1},β), $Re_{d_1} = \frac{\rho \overline{U} d_1}{\mu} = \frac{\overline{U} d_1}{\nu} = \frac{4Q}{\pi d_1 \nu}$

K_o=CE= flow coefficient= f(Re_{d1},β), see Fig. 10.6 & Fig. 10.11

the pressure drop using a manometer $\Delta p = (\gamma_m - \gamma)H$

$$A_o = \frac{\pi d_o^2}{4}$$

Compressibility effect

$$Q = Q_I Y = CEA_0 Y \sqrt{\frac{2\Delta p}{\rho_1}}$$

Y= expansion factor=f(Re_{d1},β), see Fig. 10.7

Compressibility effect is considered when $(p_1 - p_2)/p_1 \geq 0.1$

For Venturi meter

$$2*10^5 \leq Re_{d1} \leq 2*10^6$$

$$0.4 \leq \beta \leq 0.75$$

for cast unit C=0.984

for machine units C=0.995

Sonic nozzle $\dot{m}_{max} = \rho_1 A_o \sqrt{2RT_1} \sqrt{\frac{k}{k+1}\left(\frac{2}{k+1}\right)^{2/(k-1)}}$

where

k is the specific heat ratio

R is the gas constant in J/kg.K

ρ_1,T_1 is the upstream density and temperature

Overall pressure losses: Δp_{loss}, and power for the prime mover $\dot{W} = Q \frac{\Delta p_{loss}}{\eta}$

where η is the prime mover efficiency

Laminar flow elements $Q = \frac{\pi d^4}{128\mu} \frac{p_1 - p_2}{L}$

Vortex shedding

Strouhal number

$$St = fd / \overline{U}$$

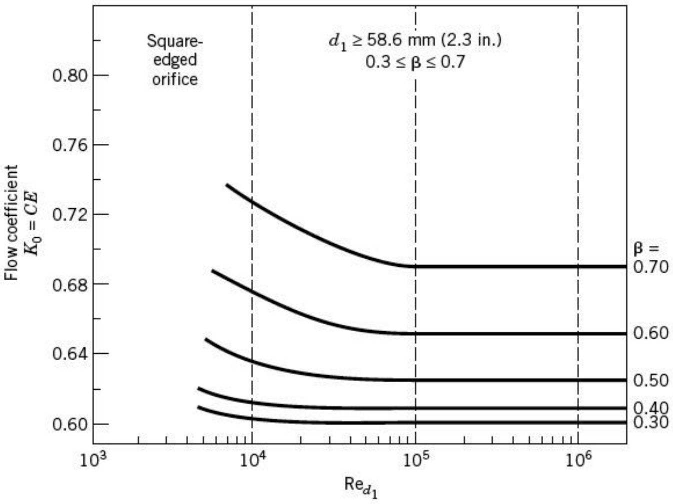
Rotameter

$$Q = AU = A\left[\frac{1}{C_d}\frac{2gV_b}{A_b}\left(\frac{\rho_b}{\rho_f}-1\right)\right]^{1/2}$$ where the flow area A is given by $A = \frac{\pi}{4}\left[(D+ay)^2 - d^2\right]$

a=tube taper=Change of diameter over change of vertical distance y

Subscript b refers to the float. Subscript f refers to fluid

A_b is the projected area of the float= $(\pi/4)d^2$. D is the inlet diameter of the meter.



$$K_0 = \frac{1}{(1-\beta^4)^{1/2}}\,(0.5959 + 0.0312\beta^{2.1} - 0.184\beta^8 + 91.71\beta^{2.5}Re_{d_1}^{-0.75})$$

Figure 10.6 Flow coefficients for a square-edged orifice meter having flange pressure taps. (Compiled from data in [2]).

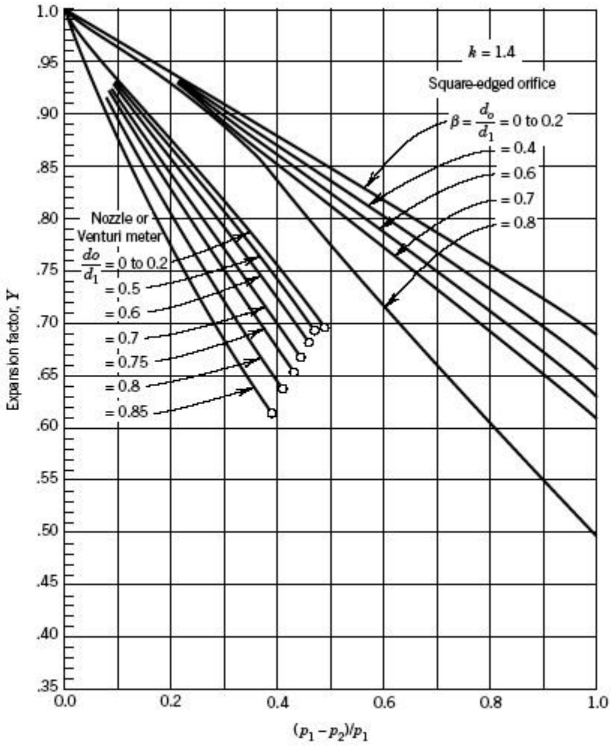
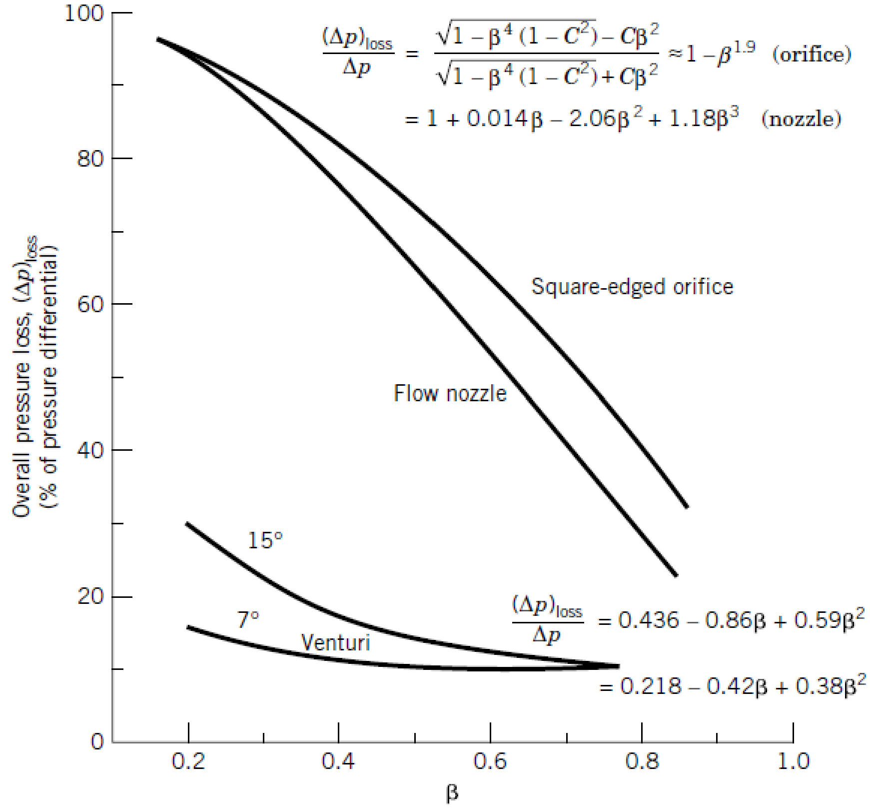


Figure 10.7 Expansion factors for common obstruction meters with $k = c_p/c_v = 1.4$. (Courtesy of American Society of Mechanical Engineers, New York; compiled and reprinted from [2].)



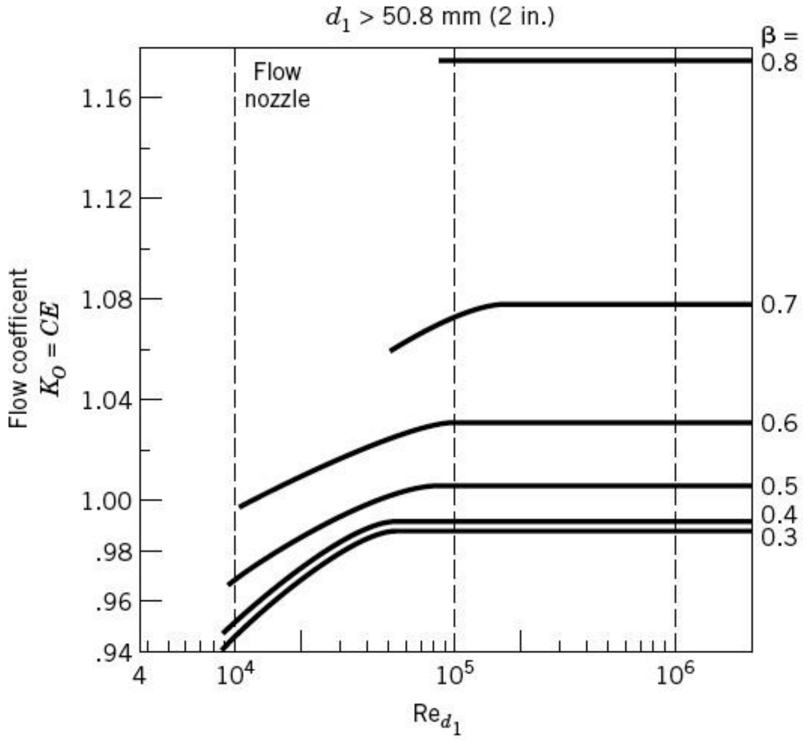
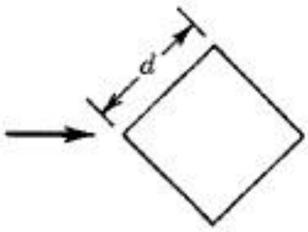
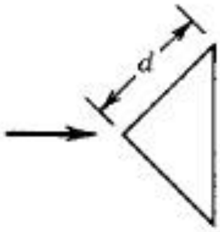
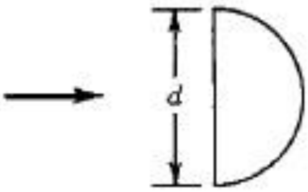
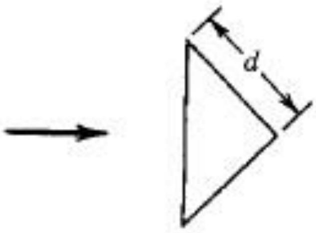
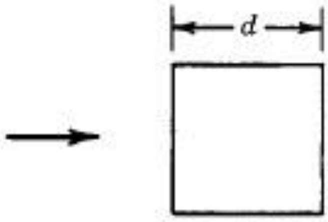


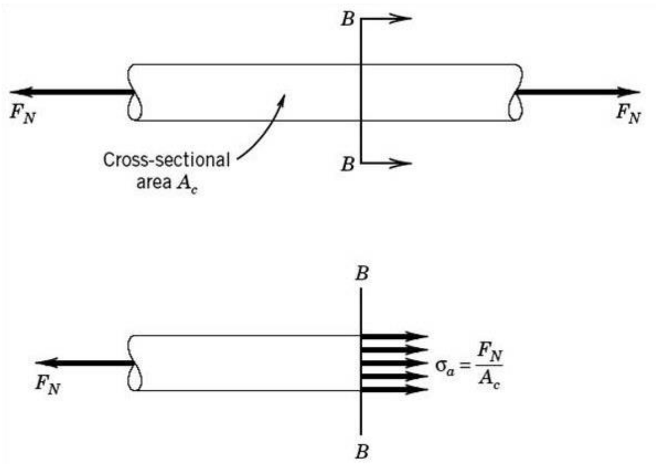
Figure 10.11 Flow coefficients for an ASME long-radius nozzle with a throat pressure tap. (Compiled from [2].)

Table 10.1 Shedder Shape and Strouhal Number

Cross Section	Strouhal Number[a]
	0.16
	0.19
	0.16
	0.15
	0.12

[a]For Reynolds number $Re_d \geq 10^4$. Strouhal number $St = fd/\overline{U}$.

Ch.11 Strain Measurements



Axial stress strain relation (Hook's law)

$$\sigma_a = E_m \varepsilon_a$$

Poission's ratio v_p

$$v_p = \frac{|\text{lateral strain}|}{|\text{axial strain}|} = \frac{\varepsilon_L}{\varepsilon_a}$$

Metallic gage

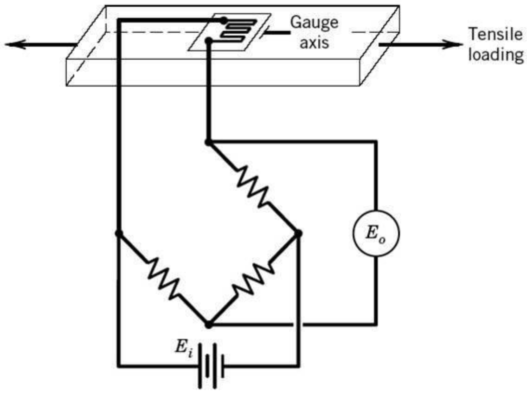
$$R = \frac{\rho_e L}{A_c}, \quad \rho_e\text{=electric resistivity}$$

$$\frac{dR}{R} = \frac{dL}{L}(1+2v_p) + \frac{d\rho_e}{\rho_e}$$

$$\frac{dR}{R} = \frac{dL}{L}(1+2v_p + \pi_1 E_m)$$

where π_1 is called piezoresistance coefficient

$$\pi_1 = \frac{1}{E_m}\frac{d\rho_e/\rho_e}{dL/L}$$



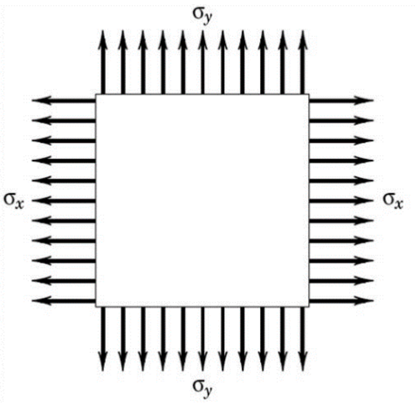
Gage factor GF is defined as

$$GF = \frac{dR/R}{dL/L} = \frac{dR/R}{\varepsilon_a}$$

Output voltage change dE_o due to bridge deflection

$$\frac{\delta E_o}{E_i} = \frac{\delta R/R}{4+2(dR/R)} \approx \frac{\delta R/R}{4} = \frac{GF\varepsilon_a}{4}$$

Strains and stresses in plan area



$$\varepsilon_y = \frac{\sigma_y}{E_m} - v_p\frac{\sigma_x}{E_m} \qquad \varepsilon_x = \frac{\sigma_x}{E_m} - v_p\frac{\sigma_y}{E_m}$$

$$\sigma_x = \frac{E_m(\varepsilon_x + v_p\varepsilon_y)}{1-v_p^2} \qquad \sigma_y = \frac{E_m(\varepsilon_y + v_p\varepsilon_x)}{1-v_p^2}$$

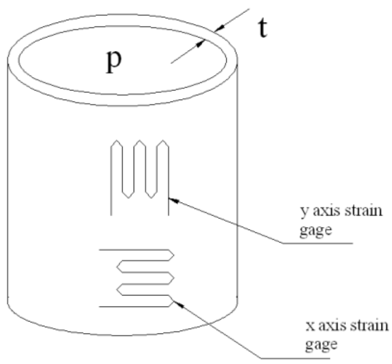
For thin walled vessels (t/r) thickness/radius <10

The relation between the pressure inside the vessel and the stresses is given by

$$\sigma_x = \frac{P\,r}{t} \qquad \sigma_y = \frac{P\,r}{2t}$$

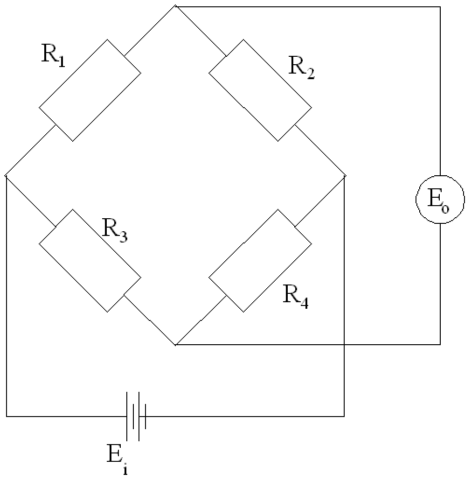
$$\sigma_x = 2\sigma_y$$

$$\varepsilon_x = \frac{\sigma_x}{E_m}(1-0.5v_p)$$



P is the pressure inside the vessel
r is the radius of the vessel
t is the vessel's wall thickness

Four arms of Wheatstone bridge



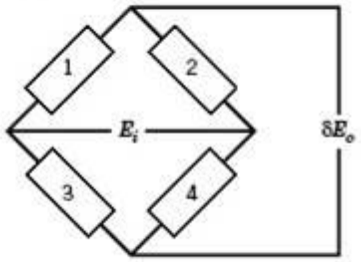
$$\frac{\delta E_o}{E_i} = \frac{GF}{4}\left(\varepsilon_1 - \varepsilon_2 + \varepsilon_4 - \varepsilon_3\right)$$

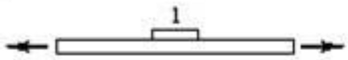
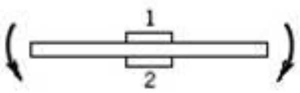
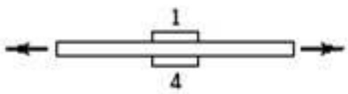
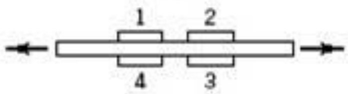
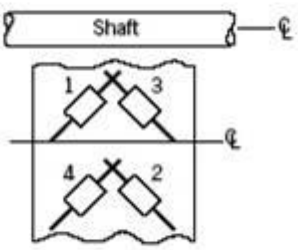
Bridge constant κ

κ =(Actual bridge output/Output of a single gauge on the bridge)

$$\frac{\delta E_o}{E_i} = \frac{\kappa GF\varepsilon}{4}$$

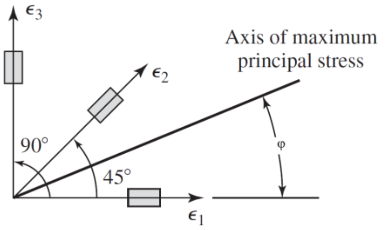
Table 11.1 Common Gauge Mountings



	Arrangement	Compensation Provided	Bridge Constant κ
	Single gauge in uniaxial stress	None	$\kappa = 1$
	Two gauges sensing equal and opposite strain—typical bending arrangement	Temperature	$\kappa = 2$
	Two gauges in uniaxial stress	Bending only	$\kappa = 2$
	Four gauges with pairs sensing equal and opposite strains	Temperature and bending	$\kappa = 4$
	One axial gauge and one poisson gauge		$\kappa = 1 + \nu$
	Four gauges with pairs sensing equal and opposite strains—sensitive to torsion only. Typical shaft arrangement.	Temperature and axial	$\kappa = 4$

Rosettes

A) 0, 45, 90 ° Rosette



$$\sigma_{\max} = \frac{E_m}{2}\left[\frac{\varepsilon_1 + \varepsilon_3}{1 - \upsilon_p} + \frac{1}{1 + \upsilon_p}\sqrt{(\varepsilon_1 - \varepsilon_3)^2 + [2\varepsilon_2 - (\varepsilon_1 + \varepsilon_3)]^2}\right]$$

$$\sigma_{\min} = \frac{E_m}{2}\left[\frac{\varepsilon_1 + \varepsilon_3}{1 - \upsilon_p} - \frac{1}{1 + \upsilon_p}\sqrt{(\varepsilon_1 - \varepsilon_3)^2 + [2\varepsilon_2 - (\varepsilon_1 + \varepsilon_3)]^2}\right]$$

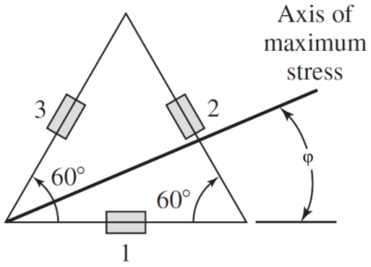
$$\tau_{\max} = \frac{E_m}{2(1 + \upsilon_p)}\sqrt{(\varepsilon_1 - \varepsilon_3)^2 + [2\varepsilon_2 - (\varepsilon_1 + \varepsilon_3)]^2}$$

The angle between the x-axis and the maximum principal stress is given by

$$\phi = \frac{1}{2}\tan^{-1}\frac{2\varepsilon_2 - (\varepsilon_1 + \varepsilon_3)}{\varepsilon_1 - \varepsilon_3}$$

φ in the first quadrant if $\varepsilon_2 > \frac{\varepsilon_1 + \varepsilon_3}{2}$, otherwise it is in the second quadrant

B) 0, 60, 120° Rosette



$$\sigma_{max}, \sigma_{min} = \frac{E_m(\varepsilon_1 + \varepsilon_2 + \varepsilon_3)}{3(1 - \nu)} \pm \frac{\sqrt{2}\,E_m}{3(1 + \nu)}[(\varepsilon_1 - \varepsilon_2)^2 + (\varepsilon_2 - \varepsilon_3)^2 + (\varepsilon_3 - \varepsilon_1)^2]^{\frac{1}{2}}$$

$$\tau_{max} = \frac{\sqrt{2}\,E_m}{3(1 + \nu)}[(\varepsilon_1 - \varepsilon_2)^2 + (\varepsilon_2 - \varepsilon_3)^2 + (\varepsilon_3 - \varepsilon_1)^2]^{\frac{1}{2}}$$

$$\varepsilon_{max}, \varepsilon_{min} = \frac{\varepsilon_1 + \varepsilon_2 + \varepsilon_3}{3} \pm \frac{\sqrt{2}}{3}[(\varepsilon_1 - \varepsilon_2)^2 + (\varepsilon_2 - \varepsilon_3)^2 + (\varepsilon_3 - \varepsilon_1)^2]^{\frac{1}{2}}$$

$$\tan(2\varphi) = \frac{\sqrt{3}(\varepsilon_3 - \varepsilon_2)}{2\varepsilon_1 - \varepsilon_2 - \varepsilon_3}$$

φ is in the first quadrant if $\varepsilon_3 > \varepsilon_2$ otherwise it is in the second quadrant

Ch. 7 Data Acquisition System

$$\text{Resolution} = \frac{V_{\max} - V_{\min}}{2^M}$$

Signal Noise ratio (SNR)

$$SNR(dB) = 20\log(2^M)$$

Sampling rate
Nyquist Theorem

$$f_s \geq 2 f_x$$

f_x=signal frequency, f_s=sampling frequency

T°C

TABLE 17 ***Type T Thermocouple*** — *thermoelectric voltage as a function of temperature (°C); reference junctions at 0 °C*

°C	0	1	2	3	4	5	6	7	8	9	10	°C
						Thermoelectric Voltage in Millivolts						
-270	-6.258											-270
-260	-6.232	-6.236	-6.239	-6.242	-6.245	-6.248	-6.251	-6.253	-6.255	-6.256	-6.258	-260
-250	-6.180	-6.187	-6.193	-6.198	-6.204	-6.209	-6.214	-6.219	-6.223	-6.228	-6.232	-250
-240	-6.105	-6.114	-6.122	-6.130	-6.138	-6.146	-6.153	-6.160	-6.167	-6.174	-6.180	-240
-230	-6.007	-6.017	-6.028	-6.038	-6.049	-6.059	-6.068	-6.078	-6.087	-6.096	-6.105	-230
-220	-5.888	-5.901	-5.914	-5.926	-5.938	-5.950	-5.962	-5.973	-5.985	-5.996	-6.007	-220
-210	-5.753	-5.767	-5.782	-5.795	-5.809	-5.823	-5.836	-5.850	-5.863	-5.876	-5.888	-210
-200	-5.603	-5.619	-5.634	-5.650	-5.665	-5.680	-5.695	-5.710	-5.724	-5.739	-5.753	-200
-190	-5.439	-5.456	-5.473	-5.489	-5.506	-5.523	-5.539	-5.555	-5.571	-5.587	-5.603	-190
-180	-5.261	-5.279	-5.297	-5.316	-5.334	-5.351	-5.369	-5.387	-5.404	-5.421	-5.439	-180
-170	-5.070	-5.089	-5.109	-5.128	-5.148	-5.167	-5.186	-5.205	-5.224	-5.242	-5.261	-170
-160	-4.865	-4.886	-4.907	-4.928	-4.949	-4.969	-4.989	-5.010	-5.030	-5.050	-5.070	-160
-150	-4.648	-4.671	-4.693	-4.715	-4.737	-4.759	-4.780	-4.802	-4.823	-4.844	-4.865	-150
-140	-4.419	-4.443	-4.466	-4.489	-4.512	-4.535	-4.558	-4.581	-4.604	-4.626	-4.648	-140
-130	-4.177	-4.202	-4.226	-4.251	-4.275	-4.300	-4.324	-4.348	-4.372	-4.395	-4.419	-130
-120	-3.923	-3.949	-3.975	-4.000	-4.026	-4.052	-4.077	-4.102	-4.127	-4.152	-4.177	-120
-110	-3.657	-3.684	-3.711	-3.738	-3.765	-3.791	-3.818	-3.844	-3.871	-3.897	-3.923	-110
-100	-3.379	-3.407	-3.435	-3.463	-3.491	-3.519	-3.547	-3.574	-3.602	-3.629	-3.657	-100
-90	-3.089	-3.118	-3.148	-3.177	-3.206	-3.235	-3.264	-3.293	-3.322	-3.350	-3.379	-90
-80	-2.788	-2.818	-2.849	-2.879	-2.910	-2.940	-2.970	-3.000	-3.030	-3.059	-3.089	-80
-70	-2.476	-2.507	-2.539	-2.571	-2.602	-2.633	-2.664	-2.695	-2.726	-2.757	-2.788	-70
-60	-2.153	-2.186	-2.218	-2.251	-2.283	-2.316	-2.348	-2.380	-2.412	-2.444	-2.476	-60
-50	-1.819	-1.853	-1.887	-1.920	-1.954	-1.987	-2.021	-2.054	-2.087	-2.120	-2.153	-50
-40	-1.475	-1.510	-1.545	-1.579	-1.614	-1.648	-1.683	-1.717	-1.751	-1.785	-1.819	-40
-30	-1.121	-1.157	-1.192	-1.228	-1.264	-1.299	-1.335	-1.370	-1.405	-1.440	-1.475	-30
-20	-0.757	-0.794	-0.830	-0.867	-0.904	-0.940	-0.976	-1.013	-1.049	-1.085	-1.121	-20
-10	-0.383	-0.421	-0.459	-0.496	-0.534	-0.571	-0.608	-0.646	-0.683	-0.720	-0.757	-10
0	0.000	-0.039	-0.077	-0.116	-0.154	-0.193	-0.231	-0.269	-0.307	-0.345	-0.383	0
0	0.000	0.039	0.078	0.117	0.156	0.195	0.234	0.273	0.312	0.352	0.391	0
10	0.391	0.431	0.470	0.510	0.549	0.589	0.629	0.669	0.709	0.749	0.790	10
20	0.790	0.830	0.870	0.911	0.951	0.992	1.033	1.074	1.114	1.155	1.196	20
30	1.196	1.238	1.279	1.320	1.362	1.403	1.445	1.486	1.528	1.570	1.612	30
40	1.612	1.654	1.696	1.738	1.780	1.823	1.865	1.908	1.950	1.993	2.036	40
50	2.036	2.079	2.122	2.165	2.208	2.251	2.294	2.338	2.381	2.425	2.468	50
60	2.468	2.512	2.556	2.600	2.643	2.687	2.732	2.776	2.820	2.864	2.909	60
70	2.909	2.953	2.998	3.043	3.087	3.132	3.177	3.222	3.267	3.312	3.358	70
80	3.358	3.403	3.448	3.494	3.539	3.585	3.631	3.677	3.722	3.768	3.814	80
90	3.814	3.860	3.907	3.953	3.999	4.046	4.092	4.138	4.185	4.232	4.279	90
100	4.279	4.325	4.372	4.419	4.466	4.513	4.561	4.608	4.655	4.702	4.750	100
110	4.750	4.798	4.845	4.893	4.941	4.988	5.036	5.084	5.132	5.180	5.228	110
120	5.228	5.277	5.325	5.373	5.422	5.470	5.519	5.567	5.616	5.665	5.714	120
130	5.714	5.763	5.812	5.861	5.910	5.959	6.008	6.057	6.107	6.156	6.206	130
140	6.206	6.255	6.305	6.355	6.404	6.454	6.504	6.554	6.604	6.654	6.704	140
150	6.704	6.754	6.805	6.855	6.905	6.956	7.006	7.057	7.107	7.158	7.209	150
160	7.209	7.260	7.310	7.361	7.412	7.463	7.515	7.566	7.617	7.668	7.720	160
170	7.720	7.771	7.823	7.874	7.926	7.977	8.029	8.081	8.133	8.185	8.237	170
180	8.237	8.289	8.341	8.393	8.445	8.497	8.550	8.602	8.654	8.707	8.759	180
190	8.759	8.812	8.865	8.917	8.970	9.023	9.076	9.129	9.182	9.235	9.288	190
°C	0	1	2	3	4	5	6	7	8	9	10	°C

TABLE 7 ***Type J Thermocouple*** — *thermoelectric voltage as a function of temperature (°C); reference junctions at 0 °C*

J °C

°C	0	1	2	3	4	5	6	7	8	9	10	°C
						Thermoelectric Voltage in Millivolts						
-210	-8.095											-210
-200	-7.890	-7.912	-7.934	-7.955	-7.976	-7.996	-8.017	-8.037	-8.057	-8.076	-8.095	-200
-190	-7.659	-7.683	-7.707	-7.731	-7.755	-7.778	-7.801	-7.824	-7.846	-7.868	-7.890	-190
-180	-7.403	-7.429	-7.456	-7.482	-7.508	-7.534	-7.559	-7.585	-7.610	-7.634	-7.659	-180
-170	-7.123	-7.152	-7.181	-7.209	-7.237	-7.265	-7.293	-7.321	-7.348	-7.376	-7.403	-170
-160	-6.821	-6.853	-6.883	-6.914	-6.944	-6.975	-7.005	-7.035	-7.064	-7.094	-7.123	-160
-150	-6.500	-6.533	-6.566	-6.598	-6.631	-6.663	-6.695	-6.727	-6.759	-6.790	-6.821	-150
-140	-6.159	-6.194	-6.229	-6.263	-6.298	-6.332	-6.366	-6.400	-6.433	-6.467	-6.500	-140
-130	-5.801	-5.838	-5.874	-5.910	-5.946	-5.982	-6.018	-6.054	-6.089	-6.124	-6.159	-130
-120	-5.426	-5.465	-5.503	-5.541	-5.578	-5.616	-5.653	-5.690	-5.727	-5.764	-5.801	-120
-110	-5.037	-5.076	-5.116	-5.155	-5.194	-5.233	-5.272	-5.311	-5.350	-5.388	-5.426	-110
-100	-4.633	-4.674	-4.714	-4.755	-4.796	-4.836	-4.877	-4.917	-4.957	-4.997	-5.037	-100
-90	-4.215	-4.257	-4.300	-4.342	-4.384	-4.425	-4.467	-4.509	-4.550	-4.591	-4.633	-90
-80	-3.786	-3.829	-3.872	-3.916	-3.959	-4.002	-4.045	-4.088	-4.130	-4.173	-4.215	-80
-70	-3.344	-3.389	-3.434	-3.478	-3.522	-3.566	-3.610	-3.654	-3.698	-3.742	-3.786	-70
-60	-2.893	-2.938	-2.984	-3.029	-3.075	-3.120	-3.165	-3.210	-3.255	-3.300	-3.344	-60
-50	-2.431	-2.478	-2.524	-2.571	-2.617	-2.663	-2.709	-2.755	-2.801	-2.847	-2.893	-50
-40	-1.961	-2.008	-2.055	-2.103	-2.150	-2.197	-2.244	-2.291	-2.338	-2.385	-2.431	-40
-30	-1.482	-1.530	-1.578	-1.626	-1.674	-1.722	-1.770	-1.818	-1.865	-1.913	-1.961	-30
-20	-0.995	-1.044	-1.093	-1.142	-1.190	-1.239	-1.288	-1.336	-1.385	-1.433	-1.482	-20
-10	-0.501	-0.550	-0.600	-0.650	-0.699	-0.749	-0.798	-0.847	-0.896	-0.946	-0.995	-10
0	0.000	-0.050	-0.101	-0.151	-0.201	-0.251	-0.301	-0.351	-0.401	-0.451	-0.501	0
0	0.000	0.050	0.101	0.151	0.202	0.253	0.303	0.354	0.405	0.456	0.507	0
10	0.507	0.558	0.609	0.660	0.711	0.762	0.814	0.865	0.916	0.968	1.019	10
20	1.019	1.071	1.122	1.174	1.226	1.277	1.329	1.381	1.433	1.485	1.537	20
30	1.537	1.589	1.641	1.693	1.745	1.797	1.849	1.902	1.954	2.006	2.059	30
40	2.059	2.111	2.164	2.216	2.269	2.322	2.374	2.427	2.480	2.532	2.585	40
50	2.585	2.638	2.691	2.744	2.797	2.850	2.903	2.956	3.009	3.062	3.116	50
60	3.116	3.169	3.222	3.275	3.329	3.382	3.436	3.489	3.543	3.596	3.650	60
70	3.650	3.703	3.757	3.810	3.864	3.918	3.971	4.025	4.079	4.133	4.187	70
80	4.187	4.240	4.294	4.348	4.402	4.456	4.510	4.564	4.618	4.672	4.726	80
90	4.726	4.781	4.835	4.889	4.943	4.997	5.052	5.106	5.160	5.215	5.269	90
100	5.269	5.323	5.378	5.432	5.487	5.541	5.595	5.650	5.705	5.759	5.814	100
110	5.814	5.868	5.923	5.977	6.032	6.087	6.141	6.196	6.251	6.306	6.360	110
120	6.360	6.415	6.470	6.525	6.579	6.634	6.689	6.744	6.799	6.854	6.909	120
130	6.909	6.964	7.019	7.074	7.129	7.184	7.239	7.294	7.349	7.404	7.459	130
140	7.459	7.514	7.569	7.624	7.679	7.734	7.789	7.844	7.900	7.955	8.010	140
150	8.010	8.065	8.120	8.175	8.231	8.286	8.341	8.396	8.452	8.507	8.562	150
160	8.562	8.618	8.673	8.728	8.783	8.839	8.894	8.949	9.005	9.060	9.115	160
170	9.115	9.171	9.226	9.282	9.337	9.392	9.448	9.503	9.559	9.614	9.669	170
180	9.669	9.725	9.780	9.836	9.891	9.947	10.002	10.057	10.113	10.168	10.224	180
190	10.224	10.279	10.335	10.390	10.446	10.501	10.557	10.612	10.668	10.723	10.779	190
200	10.779	10.834	10.890	10.945	11.001	11.056	11.112	11.167	11.223	11.278	11.334	200
210	11.334	11.389	11.445	11.501	11.556	11.612	11.667	11.723	11.778	11.834	11.889	210
220	11.889	11.945	12.000	12.056	12.111	12.167	12.222	12.278	12.334	12.389	12.445	220
230	12.445	12.500	12.556	12.611	12.667	12.722	12.778	12.833	12.889	12.944	13.000	230
240	13.000	13.056	13.111	13.167	13.222	13.278	13.333	13.389	13.444	13.500	13.555	240
°C	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**°C**

K °C

TABLE 9 ***Type K Thermocouple*** — *thermoelectric voltage as a function of temperature (°C); reference junctions at 0 °C*

°C	0	1	2	3	4	5	6	7	8	9	10	°C
						Thermoelectric Voltage in Millivolts						
-270	-6.458											-270
-260	-6.411	-6.444	-6.446	-6.448	-6.450	-6.452	-6.453	-6.455	-6.456	-6.457	-6.458	-260
-250	-6.404	-6.408	-6.413	-6.417	-6.421	-6.425	-6.429	-6.432	-6.435	-6.438	-6.441	-250
-240	-6.344	-6.351	-6.358	-6.364	-6.370	-6.377	-6.382	-6.388	-6.393	-6.399	-6.404	-240
-230	-6.262	-6.271	-6.280	-6.289	-6.297	-6.306	-6.314	-6.322	-6.329	-6.337	-6.344	-230
-220	-6.158	-6.170	-6.181	-6.192	-6.202	-6.213	-6.223	-6.233	-6.243	-6.252	-6.262	-220
-210	-6.035	-6.048	-6.061	-6.074	-6.087	-6.099	-6.111	-6.123	-6.135	-6.147	-6.158	-210
-200	-5.891	-5.907	-5.922	-5.936	-5.951	-5.965	-5.980	-5.994	-6.007	-6.021	-6.035	-200
-190	-5.730	-5.747	-5.763	-5.780	-5.797	-5.813	-5.829	-5.845	-5.861	-5.876	-5.891	-190
-180	-5.550	-5.569	-5.588	-5.606	-5.624	-5.642	-5.660	-5.678	-5.695	-5.713	-5.730	-180
-170	-5.354	-5.374	-5.395	-5.415	-5.435	-5.454	-5.474	-5.493	-5.512	-5.531	-5.550	-170
-160	-5.141	-5.163	-5.185	-5.207	-5.228	-5.250	-5.271	-5.292	-5.313	-5.333	-5.354	-160
-150	-4.913	-4.936	-4.960	-4.983	-5.006	-5.029	-5.052	-5.074	-5.097	-5.119	-5.141	-150
-140	-4.669	-4.694	-4.719	-4.744	-4.768	-4.793	-4.817	-4.841	-4.865	-4.889	-4.913	-140
-130	-4.411	-4.437	-4.463	-4.490	-4.516	-4.542	-4.567	-4.593	-4.618	-4.644	-4.669	-130
-120	-4.138	-4.166	-4.194	-4.221	-4.249	-4.276	-4.303	-4.330	-4.357	-4.384	-4.411	-120
-110	-3.852	-3.882	-3.911	-3.939	-3.968	-3.997	-4.025	-4.054	-4.082	-4.110	-4.138	-110
-100	-3.554	-3.584	-3.614	-3.645	-3.675	-3.705	-3.734	-3.764	-3.794	-3.823	-3.852	-100
-90	-3.243	-3.274	-3.306	-3.337	-3.368	-3.400	-3.431	-3.462	-3.492	-3.523	-3.554	-90
-80	-2.920	-2.953	-2.986	-3.018	-3.050	-3.083	-3.115	-3.147	-3.179	-3.211	-3.243	-80
-70	-2.587	-2.620	-2.654	-2.688	-2.721	-2.755	-2.788	-2.821	-2.854	-2.887	-2.920	-70
-60	-2.243	-2.278	-2.312	-2.347	-2.382	-2.416	-2.450	-2.485	-2.519	-2.553	-2.587	-60
-50	-1.889	-1.925	-1.961	-1.996	-2.032	-2.067	-2.103	-2.138	-2.173	-2.208	-2.243	-50
-40	-1.527	-1.564	-1.600	-1.637	-1.673	-1.709	-1.745	-1.782	-1.818	-1.854	-1.889	-40
-30	-1.156	-1.194	-1.231	-1.268	-1.305	-1.343	-1.380	-1.417	-1.453	-1.490	-1.527	-30
-20	-0.778	-0.816	-0.854	-0.892	-0.930	-0.968	-1.006	-1.043	-1.081	-1.119	-1.156	-20
-10	-0.392	-0.431	-0.470	-0.508	-0.547	-0.586	-0.624	-0.663	-0.701	-0.739	-0.778	-10
0	0.000	-0.039	-0.079	-0.118	-0.157	-0.197	-0.236	-0.275	-0.314	-0.353	-0.392	0
0	0.000	0.039	0.079	0.119	0.158	0.198	0.238	0.277	0.317	0.357	0.397	0
10	0.397	0.437	0.477	0.517	0.557	0.597	0.637	0.677	0.718	0.758	0.798	10
20	0.798	0.838	0.879	0.919	0.960	1.000	1.041	1.081	1.122	1.163	1.203	20
30	1.203	1.244	1.285	1.326	1.366	1.407	1.448	1.489	1.530	1.571	1.612	30
40	1.612	1.653	1.694	1.735	1.776	1.817	1.858	1.899	1.941	1.982	2.023	40
50	2.023	2.064	2.106	2.147	2.188	2.230	2.271	2.312	2.354	2.395	2.436	50
60	2.436	2.478	2.519	2.561	2.602	2.644	2.685	2.727	2.768	2.810	2.851	60
70	2.851	2.893	2.934	2.976	3.017	3.059	3.100	3.142	3.184	3.225	3.267	70
80	3.267	3.308	3.350	3.391	3.433	3.474	3.516	3.557	3.599	3.640	3.682	80
90	3.682	3.723	3.765	3.806	3.848	3.889	3.931	3.972	4.013	4.055	4.096	90
100	4.096	4.138	4.179	4.220	4.262	4.303	4.344	4.385	4.427	4.468	4.509	100
110	4.509	4.550	4.591	4.633	4.674	4.715	4.756	4.797	4.838	4.879	4.920	110
120	4.920	4.961	5.002	5.043	5.084	5.124	5.165	5.206	5.247	5.288	5.328	120
130	5.328	5.369	5.410	5.450	5.491	5.532	5.572	5.613	5.653	5.694	5.735	130
140	5.735	5.775	5.815	5.856	5.896	5.937	5.977	6.017	6.058	6.098	6.138	140
150	6.138	6.179	6.219	6.259	6.299	6.339	6.380	6.420	6.460	6.500	6.540	150
160	6.540	6.580	6.620	6.660	6.701	6.741	6.781	6.821	6.861	6.901	6.941	160
170	6.941	6.981	7.021	7.060	7.100	7.140	7.180	7.220	7.260	7.300	7.340	170
180	7.340	7.380	7.420	7.460	7.500	7.540	7.579	7.619	7.659	7.699	7.739	180
190	7.739	7.779	7.819	7.859	7.899	7.939	7.979	8.019	8.059	8.099	8.138	190
°C	0	1	2	3	4	5	6	7	8	9	10	°C